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Effects of Early Explicit Strategic Intervention on the Mathematics Performance of Students At-Risk for Mathematics Difficulties

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Effects of Early Explicit Strategic Intervention on the Mathematics Performance of Students At-Risk for Mathematics Difficulties

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Dedication

This dissertation is dedicated to my amazing husband and best friend Ali.

Without your love and support none of this would have been possible.

This degree is as much yours as it is mine.

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Effects of Early Explicit Strategic Intervention on the Mathematics Performance of Students At-Risk for Mathematics Difficulties

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The purpose of this study was to test the effectiveness of the early systematic strategic mathematics intervention on the mathematics performance of students at risk for Mathematics Difficulties (MD) in first grade. The investigator imbedded several intervention design features as well as learning principles into the early mathematics intervention. The features included increasing the intensity of intervention; providing explicit systematic instruction; tracking children's understanding and adjusting instruction; delivering the instruction one-on-one; and utilizing learning principles. A multiple baseline design across participants was utilized to evaluate results related to the following research questions: (1) Does the early mathematics intervention result in improved performance on a weekly proximal measure of mathematics? (2) Does the early mathematics intervention result in improved performance on a less proximal to the intervention? (3) Does the early mathematics intervention result in improved performance on a distal mathematics measure? (4) Are the effects of the intervention maintained two- and four-weeks post-intervention? (5) What are the students' perspectives on the early mathematics intervention? The first-grade students were identified at risk for MD as established by a performance at or below the 30th percentile on a standardized mathematics outcome measure. The intervention sessions were delivered four days per week, in 30–35 minutes sessions, over six weeks. The results of visual analysis and computation of the effect size of the proximal measure showed that the explicit, strategic early mathematics intervention was effective on the mathematics performance of first-grade students at risk for mathematics difficulties. All participants showed improvement in their mathematical skills and knowledge during the intervention phase and maintained intervention effects after two and four weeks. The results demonstrated that there was a significant effect of the intervention on the participants' performance in the addition-strategy task, and number-sets tasks. The result in the pre/post-intervention demonstrated significant effects of the intervention on the overall mathematical performance of first-grade students with mathematics difficulties. The result of a social validity questionnaire showed that all participants had positive perspectives toward the intervention components and agreed that the intervention had a positive impact on their understanding of mathematics.

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Chapter 1: Introduction

Statement of the Problem

According to the results from the latest Trends in International Mathematics and Science Study (TIMSS, 2015) and the Program for International Student Assessment (PISA; Dossey & Funke, 2016), the average mathematics performance of students in the U.S. is lower than the average mathematics performance of their international peers in 21 other countries (Dossey, McCrone, & Halvorsen, 2016). Students across the U.S. are not meeting standard proficiency levels or performing at the same level as their international peers. Notably, students in Asian countries and some European countries outperform U.S. students in fourth grade in an international mathematics assessment. The National Assessment of Educational Progress (NAEP) results indicated that the national fourth-grade mean scale score in mathematics performance in 2015 was 240, which was lower than that observed in 2013. Furthermore, the 2013 NAEP assessment demonstrated that students in the U.S. are not succeeding in mathematics (National Center for Education Statistics, 2013); thus, there is concern about the overall mathematics performance of U.S. students (National Center for Education Statistics, 2009).

Approximately five to eight percent of school-age students are identified as having mathematics learning disabilities (Geary, 2011), which can have long-term consequences as students move through the grades and encounter more difficult curricula (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Jordan, Glutting, & Ramineni, 2010). As early as kindergarten and first grade, achievement gaps are present between the average students and those who enter school with a poor understanding of mathematics (Strand Cary et al., 2017). This is alarming because students acquire foundational skills in the early grades. Researchers have found that students with mathematics difficulties (MD) in early grades demonstrate procedural errors and immature counting strategies, e.g. using their fingers to solve basic addition and subtraction facts more than their typically-developing peers (Geary, 2011; Shrager & Siegler, 1998). Additionally, they have developmental delays in numerical knowledge and have persistent problems with quick retrieval of basic facts (Russell Gersten, Jordan, & Flojo, 2005).

Findings from numerous studies demonstrate early numeracy knowledge at school entry is the strongest predictor of future academic success in mathematics and in other academic domains (Duncan et al., 2007). For example, children who leave kindergarten below the 10th percentile in mathematics have a 70% chance of remaining at this level five years later and likely will be classified as having mathematics learning disabilities (Morgan, Farkas, & Wu, 2009). Also, students who are disadvantaged upon entering kindergarten and do not receive high-quality mathematics instruction and support may continue to experience lower achievement in mathematics throughout their later elementary grades. For example, students from disadvantaged backgrounds, students with disabilities and English learners fail to achieve at desired or even basic levels in primary grades (National Assessment of Educational Progress, 2015).

EARLY MATHEMATICS INTERVENTIONS FOR STUDENTS WITH MATHEMATICS DIFFICULTIES

Early mathematics interventions for at-risk students have been developed and tested for efficacy (e.g., Jordan, Glutting, Dyson, Hassinger-Das, et al., 2012; Klein et al., 2008). There are a few narrative reviews on the early numeracy interventions (e.g. Mononen, Aunio, Koponen, & Aro, 2014; Raghubar & Barnes, 2017; Wang, Firmender, Power, & Byrnes, 2016). Wang et al. (2016) found an overall moderate Effect Size (ES) for early mathematics interventions. However, in most of these interventions for children with or at high risk for MD, a significant subsample of students showed minimal response to the interventions. Several decades of studies of interventions for children with or at risk for MD, including systematic review of interventions in the domain of mathematics, have shown that there are several intervention design features associated with positive ES (Fletcher & Vaughn, 2009; R. Gersten et al., 2009; Wang, Firmender, Power, & Byrnes, 2016). These instructional design features include but are not limited to increasing the intensity of instruction in mathematics in addition to Tier 1 instruction; providing explicit, systematic instruction that integrates developmental research in mathematics with principles of direct instruction; cumulative review; teaching mathematical concepts to mastery; use of concrete and visual representations including manipulatives; and tracking children's understanding and adjusting instruction (R. Gersten et al., 2009). Also, Wang et al. (2016) indicated in their meta-analysis that there was a tendency for larger effects when the program presented content to children individually (Wang et al., 2016).

Over the last few decades, researchers and educators have been identifying and evaluating solutions to improve mathematics performance in children who respond minimally to current interventions. These solutions include intensifying interventions; explicitly teaching for transfer; increasing the comprehensiveness of taught skills and strategies; and addressing the cognitive limitations of students with learning disabilities (e.g., D. Fuchs, Fuchs, & Vaughn, 2014). Booth et al. (2017) have identified learning principles that could be integrated into mathematics interventions in order to improve mathematics performance (Booth et al., 2017). Several of these learning principles have been tested in applied learning studies in both children and adults in mathematics (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). These principles include scaffolding, practice testing, self-explanation, multiple representations, analogical comparison, error reflection, worked examples, and feedback (Dunlosky et al., 2013; Geary, Berch, Ochsendorf, & Koepke, 2017). There is evidence for the effectiveness of the learning principles in promoting mathematics learning in typically-developing students. The fact that these principles are prominent in countries that consistently outperform the U.S. suggests these principles may also be useful in U.S. classrooms, especially for students with MD. Nozari et al. (2018) conducted a meta-analysis of early mathematics interventions to explore the overall effects of early mathematics interventions that have been conducted in the U.S. and investigated the possible moderating effects of integrating learning principles into the design of early mathematics interventions. The authors found an overall moderate ES (mean ES = 0.54) and indicated that there was a tendency for an intervention to produce larger effects when it included error reflection and practice testing ($p < .05$).

EARLY MATHEMATICS CONCEPTS

Mathematics is a broad concept that includes basic number knowledge (cardinality, ordinality, one-to-one correspondence, and number estimation), number and operations in base ten (single-digit and multi-digit calculations), operations and algebraic thinking (solving addition and subtraction word problems), measurement and data, and geometry. Multistep mathematics tasks require conceptual understanding, calculation of intermediate values, and integration of different sources of knowledge. Conceptual understanding helps students organize mathematical information into a coherent whole, improve retention of mathematical knowledge, and provides students with the ability to connect concepts and procedures (Geary et al., 2017). Teachers in the U.S. are required to cover various mathematics topics, without going into depth and developing a concrete understanding of mathematical ideas for students (Schmidt & Houang, 2012). This leaves students—especially those entering kindergarten with low mathematical knowledge—at risk of not receiving sufficient exposure and fully developing an understanding of the most critical concepts of number, including understanding relations between numbers and the ability to manipulate numbers to solve mathematical problems. The fundamental mathematics concepts and skills that children need to be taught in early grades include counting and cardinality, operations and algebraic thinking, and number and operations in base ten.

Counting and cardinality

Counting is the first formal introduction to numbers and sets the foundation for many of the mathematical skills that children must master in Kindergarten through second grade. For example, children use counting and their understanding of cardinality to solve simple addition and subtraction problems (e.g., in solving 4 + 3, a student may start with four and count three more to reach an answer of seven). Understanding of counting and cardinality also sets the stage for other operations such as mastering multiplication and division in the later grades (Common Core State Standards for Mathematics [CCSSM], 2013) Difficulties in counting skills are related to one of the three principles: one-to-one correspondence, in which one counting tag is applied to each object; ordinality, in which number tags must be applied in an invariant order; and cardinality, which is the number of elements in the set (Gelman, 2006)

Operations and algebraic thinking

Operations and algebraic thinking require understanding addition as putting together and adding to and understanding subtraction as taking apart and taking from (CCSSM, 2013). Students should be able to solve addition and subtraction word problems by using objects or drawings to represent the problem. For example, in first grade, students are taught to decompose numbers less than or equal to 10 into pairs in more than one way and use multiple methods to record their solutions (by equations, e.g. 2 + 3 = 5 and 5 = 1 + 4, or drawing). Manipulating numbers to make problems easier to solve is a complex skill for students to learn and requires a deep conceptual understanding of number (Jordan, Glutting, Dyson, Hassinger-Das, et al., 2012).

Number and operations in base ten

Understanding skills related to number and operations in base ten requires students to work with numbers 11 through 19 and gain foundations for place value, for example, composing and decomposing numbers from 11 to 19 into ten ones and some further ones (CCSSM, 2013). The base ten system provides a way for individuals to represent an infinite amount of numbers by stringing together only 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Also, place value gives meaning to digits depending on where they fall within the string of digits. Understanding base ten is a critical skill for students to master in order to understand the organizational structure of our number system.

Statement of Purpose

Early numeracy is an umbrella term that includes skills such as counting aloud, knowing the number symbols, recognizing quantities, discerning number patterns, comparing numerical magnitudes, and manipulating quantities. Early numeracy knowledge at school entry is the strongest predictor of future academic success in mathematics (Duncan et al., 2007; Morgan, Farkas, & Qiong Wu, 2009). Thus, early intervention for such at-risk students is needed. Despite the improvement of mathematics interventions for the early grades, many young students with MD do not benefit from these interventions. In this dissertation, the investigator embedded several intervention design features as well as learning principles into the early mathematics intervention. These features included increasing the intensity of intervention; providing explicit systematic instruction; tracking children's understanding and adjusting instruction; delivering the instruction one-on-one, and utilizing several learning principles (e.g., practice testing, using multiple representations, error reflection, and scaffolding). The purpose of this study was to test the effectiveness of the early mathematics intervention on the mathematics performance of students at risk for MD in first grade. The investigator examined the effect of an early systematic strategic mathematics intervention that included several learning principles.

In summary, the following research questions guided this study:

1. Does the early mathematics intervention result in improved performance on the TEMI-AC, which is a weekly proximal measure of mathematics?
2. Does the early mathematics intervention result in improved performance on the Number System Knowledge (NSK) tasks, which is less proximal to the intervention?
3. Does the early mathematics intervention result in improved performance on the TEMA, which is a distal mathematics measure?
4. Are the effects of the intervention maintained two- and four-weeks post-intervention based on performance on the TEMI-AC?
5. What are the student's perspectives on the early mathematics intervention?

Chapter 2: Review of Literature

This review of the literature summarizes key research in the field surrounding early mathematics, students at risk for MD, and learning principles. First, the rationale for providing early interventions to students at risk for MD is discussed. Following this section, research on early mathematics interventions with high impact is described and the learning principle in each intervention study is summarized. Finally, the importance of each learning principle in early mathematics learning is discussed. These sections emphasize how the research relates to the mathematics intervention for students at risk for MD in this dissertation.

RATIONALE

Students with MD struggle to develop the sufficient number sense knowledge and skills required to facilitate later conceptual understanding of mathematical concepts and skills including fact and computation (Locuniak & Jordan, 2008). Typically-developing students achieve high number sense skills by first grade, but students with MD have been shown to have continuing deficits in these skills through third grade (Desoete & Grégoire, 2006). About five to eight percent of school-aged individuals experience persistently low mathematics achievement and require more intensive intervention (Geary, 2011; Murphy, Mazzocco, Hanich, & Early, 2007). The NAEP (2015) results indicated that students with MD demonstrated the greatest lags in basic mathematics skills. For example, results showed that the overall population of students with disabilities who scored below the basic level has increased from 43% in 2005 to 45% in 2015 (NAEP, 2015).

Students who are disadvantaged upon entering kindergarten and do not receive high-quality mathematics instruction and support may continue to experience lower mathematics achievement throughout their later elementary grades. For example, students from disadvantaged backgrounds, students with disabilities and English language learners fail to achieve at desired or even basic levels in primary grades (NAEP, 2015). It behooves researchers and educators to identify effective early mathematics interventions that teachers can utilize for instructing these children who demonstrate poor mathematics outcomes.

This dissertation integrates multiple learning principles into the design of an early mathematics intervention for supporting the development of early numeracy knowledge and skills in children at risk for MD. Learning principles have been used in previous early interventions for this population of students and have been shown to be effective in promoting skills in areas of counting and cardinality, addition and subtraction combination, magnitude comparison, and place value (e.g., Clarke et al., 2015; R. Gersten et al., 2015; Hassinger-Das, Jordan, & Dyson, 2015).

LEARNING PRINCIPLES IN MATHEMATICS INTERVENTION

Cognitive and educational psychologists and researchers have been identifying and developing learning principles that teachers can use in classrooms to improve conceptual understanding and retention of information across mathematics. The learning principles are inexpensive to implement in the classroom and have been shown to improve mathematics learning (Dunlosky et al., 2013). They include elaborative interrogation, practice testing, interleaving practice, scaffolding, feedback, error reflection, multiple representations, and worked examples (Dunlosky et al., 2013; Geary et al., 2017). In the following sections, the instructional implication of each learning principle in the mathematics domain is described.

Practice testing

Practice testing refers to any form of testing that students can engage in independently. For example, practice testing involves practicing recall of addition and subtraction fact via the use of actual or virtual flashcards, completing practice problems or questions included at the end of mathematics textbooks, or completing practice tests included in electronic supplemental materials. Carpenter (2009) proposed that testing can improve retention by triggering elaborative retrieval processes. Attempting to retrieve target information involves a search of long-term memory that activates related information and facilitates learning (Carpenter, Pashler, & Cepeda, 2009).

Elaborative interrogation

Elaboration is a abroad concept that means adding more information by prompting the learner to generate an explanation for an explicitly stated fact (Dunlosky et al., 2013; Ozgungor & Guthrie, 2004). By asking question that can be answered using information in the lesson, teachers can prompt students to integrate new information from a written lesson with other information from the lesson or with their prior knowledge (Smith, Holliday, &

Austin, 2010). These explanatory questions include, but are not limited to, 'Why does it make sense that…?', 'Why is this true?', and simply 'Why?'

Interleaving practice

The interleaving principle suggests that when practice problems are alternated, with a problem on one concept followed by a problem on another concept, students learn better than if problems are blocked or grouped by concept (Rohrer, 2012). According to this principle, spreading out learning opportunities causes better long-term retention of information than providing multiple learning opportunities one right after the other (i.e., massed practice; Dunlosky et al., 2013). Students benefit from interleaving practice in the retrieval of both addition and multiplication facts (Schutte et al., 2015). Pellegrino (2012) suggested that students who received practice on a set of previously learned relevant concepts performed better at the post-test that assessed understanding of the new content. Interleaving practice can help students to build a strong relationship between problem types and appropriate solution strategies (Rohrer, Dedrick, & Burgess, 2014). It also allows more opportunities for students to identify errors and refine their knowledge on several different mathematical content areas including fractions, addition, and equation-solving (Li, Cohen, & Koedinger, 2012).

Scaffolding

Scaffolding emerged from Vygotsky's (1978) zone of proximal development (ZPD), which is the distance between the learners' current development and their potential development. Vygotsky suggested that adults play a role in children's problem-solving activities by considering the child's level of understanding (Kupers, van Dijk, & van Geert, 2015). Scaffolding involves three components: (1) students' level of understanding, (2) withdrawal of scaffolding, and (3) transferal of responsibility from the teacher or adult to students. For students who are at risk for MD, the scaffolding through a short-term validated intervention creates a strong foundation to experience long-term success with their mathematics schooling (L. Fuchs, Fuchs, & Compton, 2012).

Error reflection

This principle emerges from cognitive dissonance theory that is a state of tension or discomfort that arises whenever one holds two cognitions that are inconsistent with one another (Festinger, 1957). In other words, inconsistency between two cognitions creates an aversive state similar to hunger or thirst that gives rise to a motivation to reduce the inconsistency. Tenets of dissonance theory suggest that learning from errors can be most effective if learners are encouraged to identify the features of the problem that caused them to make an error (i.e. find their mistake). Error reflection could be particularly relevant to mathematics because students' reflection on errors (either their own errors or other learners' errors) leads to better understanding (Siegler & Chen, 2008). A number of research studies have shown that reflecting on errors is effective for mathematics learning. For example, Durkin and Rittle-Johnson (2012) have found that students benefit from comparing correct and incorrect examples in learning decimal magnitudes. Also, studying and explaining errors can be beneficial in mathematics learning for both students with low and high prior knowledge (e.g., Barbieri & Booth, 2016; Heemsoth & Heinze, 2014)

Multiple representation

Studies have shown that different types of representation such as concrete, semi-concrete and abstract have unique benefits and students learn most effectively by making connections between different representations (Pachler et al., 2007). Within the domain of mathematics, researchers have suggested that concrete representations support conceptual understanding (e.g., Kaminski, Sloutsky, & Heckler, 2008) and abstract representations can support transfer (Bock, Deprez, Dooren, Roelens, & Verschaffel, 2011). Therefore, integrating concrete and abstract representations may provide substantial benefits.

Worked examples

This principle suggests that asking learners to study examples of worked-out solutions to problems is more effective than asking them to solve all of the problems themselves (Sweller & Cooper, 1985). Worked examples help students to explain the steps in the example, connect new information to prior knowledge, and generate inferences to fill knowledge gaps (Mayer, 2014). Moreover, studying and explaining worked examples can be beneficial for both novice and expert learners (Booth, Lange, Koedinger, & Newton, 2013). Studies showed that explaining worked examples improves conceptual understanding in learning both algebra and geometry (Booth et al., 2013; Reed, Corbett, Hoffman, Wagner, & MacLaren, 2013).

Early Mathematics Intervention Studies and Learning Principles

Early mathematics is an umbrella term that encompasses several skills such as counting aloud, knowing the number symbols, recognizing quantities, discerning number patterns, comparing numerical magnitudes, and manipulating quantities. Many longitudinal studies (e.g., Chard et al., 2005; Clarke & Shinn, 2004; Jordan, Kaplan, Ramineni, & Locuniak, 2009) have shown that early mathematics knowledge and skills in lower grades is a powerful predictor of later mathematics achievement (Raghubar & Barnes, 2017). In this section, the integration of the learning principles in recent early mathematics interventions with high impact are summarized. The magnitude of effect sizes of these early mathematics intervention studies varied from moderate to large.

Cary et al. (2017) investigated the impact of the Fusion program on the mathematics achievement of at-risk students. Fusion is a 60-lesson, Grade 1 (Tier 2) mathematics intervention that is designed to promote students' mathematical proficiency with whole number concepts and skills. Cary et al. (2017) used multiple learning principles in the design of Fusion including scaffolding, feedback, error reflection, self-explanation, and multiple representations. Each lesson contains multiple elements for interventionists to: (a) model what students are to learn; (b) assist students as they work as a group or individually through instructional scaffolding; (c) facilitate opportunities for students to engage in mathematical discourse; and (d) provide specific academic feedback (e.g., including error corrections and reasons for correct responses) to students during the mathematics activities. The intervention also included student mathematics verbalizations and multiple representations of mathematics concepts. For students struggling with mathematics, verbalizations permit a structured opportunity to communicate mathematical understanding, thinking, and reasoning. The intervention incorporated a variety of mathematics representations, including number lines, strip diagrams, and place value blocks, to build a deep understanding of whole numbers and operations. These representations were scaffolded across concepts and systematically withdrawn to promote abstract mathematical thinking. Results suggested a positive, statistically significant effect for the proximal ProFusion measure (researcher-developed) with Hedges's $g = 0.60$.

Clarke et al. (2017) reported the use of multiple learning principles in the ROOTS intervention. ROOTS is a Tier 2 kindergarten program that consists of 50 lessons designed to build students' whole number proficiency. The intervention included explicit teacher modeling, deliberate practice, multiple representations of mathematics, and academic feedback. It also provided frequent opportunities for students to verbalize their mathematical thinking and discuss problem-solving methods. The authors found statistically significant differences by the condition in gains from fall to spring for three dependent variables including assessment of early numeracy skills (Hedges's $g = 0.75$), assessing student proficiency in early number sense (Hedges's $g = 0.52$) and test of early mathematics ability (Hedges's $g = 0.25$).

Gersten et al. (2015) utilized the Number Rocket intervention to promote number knowledge in students with MD in early grades. Number Rocket is a scripted intervention program that is teacher-directed and is delivered to groups of two or three students. Gersten et al. (2015) used six learning principles in the design of the Number Rocket intervention:

self-explanation, practice testing, distributed practice, scaffolding, feedback, and error reflection. The authors reported that in the treatment condition, teachers presented clear models and provided opportunities for students to practice each new concept, strategy or skill extensively. In addition, students received immediate feedback on whether their responses were correct. In the treatment condition, cumulative reviews were provided, and ideas and operations taught in one unit were integrated into appropriate segments of subsequent lessons. Each unit began with a brief individual quiz that reviewed the content of the previous unit. The content covered grade-level material and some relevant material from kindergarten. Students taught with the intervention showed significantly superior performances on a broad measure of mathematics proficiency.

Fuchs et al. (2013) compared the effects of two practice conditions and a comparison condition. Both practice conditions occurred on the same content and provided immediate corrective feedback, abstract and concrete representations, error reflection and scaffolding in number knowledge. The only difference between the treatment conditions was the two contrasting forms of practice that students received. The first group received non-speeded practice that reinforced the thoughtful application of the relations and principles that serve as the basis of reasoning strategies to support fact retrieval. On the other hand, the second group received speeded practice that promoted quick responding and use of efficient counting procedures to generate many correct answers and form long-term representations to support retrieval. The findings suggested that the intervention with speeded practice or non-speeded practice was effective in four mathematics measures, including enhancing arithmetic (ES = 0.87 for speeded practice, and ES = 0.38 for non-speeded practice), complex calculations (ES = 0.69 for speeded practice, and 0.49 for non-speeded practice), number knowledge (ES = 0.29 for speeded practice, 0.19 for non-speeded practice), and word-problem learning (ES = 0.29 speeded practice, 0.27 for non-speeded practice). Therefore, the treatment group condition with speeded practice outperformed the treatment group with non-speeded practice (ES = 0.51).

Clarke et al. (2011) conducted an intervention study, in which teachers provided models or think-aloud strategy as they solved problems. Then, students solved similar problems and teachers provided specific and immediate feedback as students verbalized and explained their solutions and understanding of the underlying mathematical concepts. Finally, the intervention contained frequent and cumulative reviews, including the use of the distributed practice of key concepts to ensure retention and generalization of acquired mathematics knowledge. The students in the treatment condition outperformed students in the control condition on early numeracy curriculum-based measures (Hedges's $g = 0.52$).

Dyson et al. (2011) conducted a number sense intervention for kindergartners from low-income families. The intervention emphasized whole number concepts related to counting, comparing, and manipulating sets. The intervention included multiple representations (primarily chips, black dots, and fingers) and centered activities on a number list from one to 10 to help children understand the number concept. Students reviewed the skills and concepts incrementally over the course of the 24 lessons. They received a cumulative review based on the needs of individuals in the group. For example, if one child needed extra practice with recognition of numbers one through 10, the instructor worked with this child while the other children worked on number activity sheets. The researchers used a compare and contrast approach throughout the activities. For example, opposites such as before and after, addition and subtraction, n + 1 and n – 1 were presented simultaneously. Also, the research team monitored children's progress with informal assessments throughout the lessons to address students' individual needs. The intervention group made meaningful gains relative to the control group at immediate as well as delayed post-test on a measure of early numeracy. Intervention children also performed better than control groups on a standardized test of mathematics calculation at immediate post-test.

Sood and Jitendra (2011) examined the effectiveness of the number sense instruction on kindergarten students' number proficiency using a variety of representations (e.g., dot cards, counters, cubes, and five- and ten-frames) and instructional activities that integrated numbers and quantities into real situations. Number sense instruction is a multicomponent intervention that emphasizes relationships among numbers one through 10 (spatial relationships, one more, one less, two more, and two less, benchmarks of five and ten, and part-part-whole relationships). Teachers provided children with a range of relevant experiences and used teaching strategies to develop mathematical concepts and integrate mathematics with other activities. Teachers also provided modeling and scaffolding in developing number sense and provided students with feedback and guidance, as needed. Results indicated significant differences favoring the treatment group on all measures of number sense at post-test and on a three-week retention test.

Finally, Bryant et al. (2011) integrated early numeracy intervention self-explanation, scaffolding, multiple representations (e.g., base ten models, connecting cubes, number lines, ten-frames, hundred charts, and fact cards), guided practice, independent practice, error reflection, opportunities for meaningful practice examples, and review. Effects were significantly stronger for tutoring compared with a no-tutoring control group on simple arithmetic (ES = 0.55), place value (ES = 0.39), and number sequences (ES = 0.47).

SUMMARY

Across the intervention studies for young children with MD, interventions contained between three and eight cognitive learning principles, and intervention studies that included more learning principles showed larger effect sizes. The majority of early mathematics intervention studies (87%) reported using practice testing to monitor student progress or review the previous lessons (e.g., Dyson, Jordan, & Glutting, 2013b; Jordan, Glutting, Dyson, Hassinger-Das, et al., 2012; Klein et al., 2008). There were two studies (Clarke et al., 2011; R. Gersten et al., 2015) that showed evidence of using interleaving practice. For example, Clarke et al. (2011) stated that each intervention lesson contained four to five activities, each of which focused on one of three content areas: number and operations, measurement, and geometry. The first activity introduced a mathematics concept or skill that was central to the lesson's overall objective. The second and third activities included either an extension of the first activity or a review of previously learned material. The fourth activity targeted previously learned material from a different content area. Therefore, children received continuous practice in different content areas (Clarke et al., 2011).

Several intervention studies for children with or at risk of learning disabilities have shown that integration of these learning principles, such as worked examples, interleaving, and error reflection, can positively affect students' outcomes in mathematics (Barbieri & Booth, 2016; Kao, Davenport, & Matlen, 2007; Rohrer, Dedrick, & Burgess, 2014). Dunlosky et al. (2013) suggested that using these principles helps students to better regulate their learning with effective techniques. In this dissertation, the investigator proposes that the integration of learning principles into early mathematics interventions can facilitate mathematics learning in students who are at risk for MD. The investigator explicitly embedded the learning principles into the design of intervention to increase responsiveness to intervention.

Chapter 3: Method

The purpose of this study was to determine the effectiveness of an early mathematics intervention on the mathematics performance of first-grade students who were at risk for MD as established by a performance at or below the 30th percentile on a standardized mathematics outcome measure. The intervention sessions were delivered four days per week, in 30–35 minutes sessions, over six weeks. The effects of an explicit, systematic, strategic early mathematics intervention on the mathematics performance of first-grade students at risk for MD was tested. This study was guided by the following research questions:

1. Does the early mathematics intervention result in improved performance on the TEMI-AC, which is a weekly proximal measure of mathematics?
2. Does the early mathematics intervention result in improved performance on the NSK task, which is less proximal to the intervention?
3. Does the early mathematics intervention result in improved performance on the TEMA, which is a distal mathematics measure?
4. Are the effects of the intervention maintained two- and four-weeks post-intervention based on performance on the TEMI-AC?
5. What are the students' perspectives on the early mathematics intervention?Note: RQ = Research Question; TEMI-AC = Texas Early Mathematics Inventory–Aim Checks (University of Texas System/Texas Education Agency, 2009); TEMA-3 = Test of Early Mathematics Ability-3 (Ginsburg & Baroody, 2003).

Table 3. 1 shows the alignment of the research questions to dependent variables and measures.

Research Question	Dependent Variables	Measures
RQ 1: Student Outcomes Does the early mathematics intervention result in improved performance on the TEMI-AC, which is a weekly proximal measure of mathematics?	Mathematics performance on weekly proximal measure of early numeracy skills	TEMI–AC administered weekly
RQ 2: Student Outcomes Does the early mathematics intervention result in improved performance on the NSK task, which is less proximal to the intervention?	Students' performance on NSK measures	Re-administration of addition-strategy task, number sets task, and number line estimation task, post-intervention
RQ 3: Maintenance Does the early mathematics intervention result in improved performance on the TEMA, which is a distal mathematics measure?	Mathematics performance on early numeracy proximal measure	TEMI–AC (administered two- and Four-weeks post-intervention)
RQ 4: Generalization Are the effect of the intervention maintained two- and four-weeks post-intervention based on performance on the TEMI-AC?	Mathematics performance on early numeracy distal measure	Re-administration of TEMA-3, post-intervention
RQ 5: Social Validity What are the students' perspectives on the early mathematics intervention?	Social validity of the intervention	Researcher developed social validity survey containing rating scales

Note: RQ = Research Question; TEMI-AC = Texas Early Mathematics Inventory–Aim Checks (University of Texas System/Texas Education Agency, 2009); TEMA-3 = Test of Early Mathematics Ability-3 (Ginsburg & Baroody, 2003).

Table 3. 1: Research Questions, Dependent Variables, and Measures

University and school IRB approval, as well as parent or guardian consent for potential participants, were obtained prior to the start of the study. The investigator explained to the participants the purpose of study to obtain informed assent. The school administrators were provided with the aims of the study and contacts were established with the first-grade teachers.

PARTICIPANTS

Participants were recruited from a public elementary school in Austin Independent School District (AISD). Potential participants were identified through the school's universal screener and a norm-referenced measure to determine if they qualified for the study. Students' demographics were collected from the school on one occasion in the fall semester. The requested demographics variables included the following: gender, date of birth, primary disability code, secondary disability code, special education indicator code, home language code, and result of the universal screening measures at school (TEKS-aligned Pearson Education End of Year test; Envisions, 2014). Table 3. 2 provides the demographic data for the participants.

Screening Criteria

In this study, multiple-gating procedures were utilized as cost-effective stepwise screening mechanisms to identify eligible participants (Loeber, 1990; Loeber, Dishion, & Patterson, 1984). Using a multiple-gating approach, the investigator used two successive steps to identify eligible students. The first step was using the results of the school-administered Texas Essential Knowledge and Skills (TEKS) aligned Pearson Education End of Year test (Envisions, 2014) from the school to identify students whose scores fell below the proficiency level (70% accuracy on the test as designated by DMAC Solutions, Education Service Center, 2018). The second step was utilizing the standardized Test of Early Mathematics Ability, Third Edition (TEMA-3; Pro-Ed) for those students who were below the proficiency level on the Pearson Education End of Year test to identify students whose scores fell at or below the 30th percentile from the pool of students identified through the initial universal screening procedures. From students whose scores fell at or below the standard score of 90 (30th percentile) on TEMA-3, the four students with the lowest scores were eligible for the intervention. Purpura et al. (2015) used sensitivity (i.e., the proportion of students correctly classified as at risk) and specificity (i.e., the proportion of students correctly classified as not at risk) to identify ideal cutoff scores on TEMA-3 (Purpura & Logan, 2015) The researchers identified students at risk of later mathematics difficulties based on TEMA-3 scores of 90 or below. Researchers also suggested over-identifying children at risk of later difficulties and then removing false positives through subsequent assessment methods, rather than under-identifying and missing children in need of further instruction as at-risk of mathematics difficulties (Purpura & Logan, 2015).

Students were excluded if they were not available to participate in the intervention during specific time periods of the school day or after school, or if they did not have their parents' permission. Also, students were excluded if they had limited English proficiency. The investigator identified four students to participate in the study based on inclusion and exclusion criteria.

SETTING

The study took place in a library at a public elementary school located in central Texas. The school serves 310 students in pre-K through fifth grade. School demographics were acquired from the school website for the 2018–2019 school year. Demographics of the student population were as follows: 66.1% were Hispanic; 17.4% were White; 10.2% were African American; 1.3% were Asian; 4.6% were two races; 0.3% were American Indian; and 0.5% were Pacific Islander. Most of the students (55.2%) were economically disadvantaged, based on free and reduced lunch status. Twenty-two percent of the students had limited English proficiency.

		Participant 1	Participant 2	Participant 3	Participant 4
Age		7	7	7	7
Grade		1	1	1	1
Gender		M	F	M	F
Ethnicity		B	H	B	H
IEP/ Disability status		None/ At-risk	None/ At-risk	None/ At-risk	None/ At-risk
F/R lunch		Y	Y	Y	Y
Universal Screener: TEKS	Row/PL	3/16	3/16	5/27	6/33
	Category	Below Average	Below Average	Below Average	Below Average
Screening: TIMA	Raw/PR	29/9	33/13	30/19	35/29
	Category	Below Average	Below Average	Below Average	Below Average

Note: F = female; M = male; H = Hispanic or Latino; B = Black or African American; PL = proficiency level; Y = yes; N = no; F/R lunch = free or reduced lunch.

Table 3. 2: Participant Demographic Information

RESEARCH DESIGN

A multiple baseline design across participants (Kennedy, 2005) was implemented to assess the effects of the explicit, systematic early mathematics intervention on the early mathematics skills of students at risk for MD, utilizing progress monitoring measures. The basis of single-case research methodology relies upon repeated measurement of dependent variables before, during, and after the introduction of the independent variable to determine if a functional relation exists (i.e., a demonstration of experimental control) (Horner et al., 2005; Kennedy, 2005). Students who meet the criterion of needing mathematics intervention were assigned to the intervention session based on availability according to the general classroom schedule. Then, they were assigned to an order in which they received the intervention based on the baseline data. For example, the first students who had stable data in the baseline received the intervention.

During the baseline phase, the four, two-minute subtests (i.e., number sequence, magnitude comparison, place value, and addition-subtraction combinations) of the Texas Early Mathematics Inventories-Aim Checks (TEMI-ACs; Texas Education Agency/ The University of Texas System, 2009) were utilized. When a stable baseline was determined for the first group, intervention sessions (30–35 min in length; administered by a trained interventionist) were provided. By starting the intervention for one student, while the remainder of the students remained in the baseline phase, one would expect to see a change in performance for the student receiving treatment but not for the students in baseline. This pattern suggests that the change in performance is likely due to the intervention and not extraneous variables (see Appendix B). In addition to the methodological rigor, a multiple baseline design allows for the measurement of change in student performance and skills when the intervention begins.

Sessions began for additional students who were identified as qualifying for this study, using multiple baseline procedures, when they achieved stable baselines on the total score of the TEMI-AC. The investigator conducted a visual analysis on the data and computed a non-parametric (NP) ES, as well as calculating Tau-U ESs across the baseline and intervention. Progress monitoring measures were administered twice per week. First-grade number-system knowledge (NSK) tasks (i.e. addition strategy tasks, number sets tasks, number line estimation tasks) and TEMA-3 were administered before and after the intervention to examine the effectiveness of the intervention on overall mathematics performance.

Independent Variable

The independent variable was the explicit, systematic early mathematics intervention, which consisted of instruction focused on addition and subtraction combinations, number sequences, magnitude comparisons, and relationships of ten lessons (see Appendix A). The intervention consisted of three major instructional components: (a) explicit instruction, (b) multiple representations, and (c) students' verbalizations. Each intervention session included a warm-up, modeled practice (interactive modeling of mathematical ideas), guided practice, independent practice, a check for understanding, and error correction. For example, Student 1 received Lesson 1 in the first intervention session. Each 30–35 minutes intervention session consisted of a warm-up (6 min), modeled practice

(8 min), guided practice (8 min), independent practice (4 min), a check for understanding, and error correction (4 min). The investigator introduced the intervention across participants based on the stability of baseline data for each student. During the intervention phase, a total of 24 sessions were provided to teach the lessons. Each session represented one lesson of the intervention. The 30 sessions occurred over the course of six weeks.

Dependent Variables

Table 3. 1 displays the dependent variables aligned with each of the research questions and the related measures.

The dependent variable for Research Question 1 (proximal outcomes) was performance on the TEMI-AC. Performance was measured through the administration of weekly progress monitoring assessments during the baseline, intervention, and maintenance phases (i.e., TEMI-AC). Researchers have frequently used the percentage of correct answers as a dependent variable in mathematics intervention studies with single-case designs (Dennis, Knight, & Jerman, 2016; Shumate, Campbell-Whatley, & Lo, 2012; Strickland & Maccini, 2013).

The dependent variable for Research Question 2 (outcomes less proximal to the intervention) was the student's score on three NSK measures including an addition strategy task, a number sets task, and a number line estimation task. Students' performances on TEMI-AC was measured two and four weeks after attending the last intervention session as the dependent variable for Research Question 3 (maintenance). The dependent variable for Research Question 4 (generalization) was participants' performance on a distal test of early mathematics knowledge and skills that were assessed through the administration of the Test of Early Mathematics Ability-3 (Hoffman & Grialou, 2005; Ryoo et al., 2015) both pre- and post-intervention.

The dependent variable for Research Question 5 (social validity) was students' responses to the social validity measure that was administered to assess perceived outcomes and effectiveness, as well as the feasibility, strengths and challenges of the intervention program (see Appendix B). The interventionist read aloud each item for the students and they chose their answer on a faces rating scale. The interventionist recorded students' answers.

MEASURES

Several math measures were used in this study. Two coding forms were also used to assess the fidelity of implementation and social validity of the intervention.

Screening and Distal Measures

Screening measures were administered once prior to the study. The screening measure was the Test of Early Mathematics Ability, Third Edition (TEMA-3), a norm-referenced measure/diagnostic tool for determining mathematical strengths and weaknesses of students, ages three through eight. The TEMA-3 was also utilized as a post-intervention distal measure. The TEMA-3 consists of 72 items in the domains of informal and formal mathematics. Informal items evaluate four domains: numbering skills, number-comparison facility, calculation skills, and understanding of concepts. Formal items evaluate numeral literacy, mastery of number facts, calculation skills, and understanding of concepts. The assessment items frequently use representations in verbal, pictorial, and written formats. Test results were reported as standard scores, percentile ranks, age and grade equivalents. Internal consistency reliabilities were all above .92; immediate and delayed alternative form reliabilities were in the .80s and .90s (Ginsburg & Baroody, 2003). Students whose scores ranked at or below the 30th percentile were qualified to participate in the study.

Number-Specific Measures

To assess NSK, three measures were administered before and after the intervention: an addition strategy task (the child was asked to solve problems on flashcards as quickly as possible without making too many mistakes), a number sets task (the child was asked to move across each line of the page from left to right without skipping any items, to circle any groups that could be put together to make the top number), and a number line estimation task (the child was asked to mark the line where the target number should lie). Geary et al. (2018) reported that the addition strategy, number set, and number line variables were highly correlated ($rs > .58$, $ps < .0001$). A confirmatory factor analysis with factor loadings constrained to equality confirmed that variables defined a single factor, $\chi2 = 0.33$, $p = .84$, goodness-of-fit index = .99 (Geary et al., 2018). These three variables were standardized ($M = 0$ and $SD = 1$), and the means of these scores were used as the score NSK outcome measure.

Addition strategy task

Fourteen simple addition problems and six more complex problems were presented, one at a time, on flashcards. The simple problems involved the integers 2 through 9, with the constraint that the same two integers were never used in the same problem (e.g., '2 + 2' were not included). The complex items were '16 + 7,' '3 + 18,' '9 + 15,' '17 + 4,' '6 + 19,' and '14 + 8.' The children were asked to solve each problem (without pencil and paper) as quickly as possible without making too many mistakes. After solving each problem, they were asked to describe how they got the answer. Based on the child's description and the experimenter's observations, each trial was classified according to problem-solving strategy (Geary et al., 2018). Problems were coded on a six-point scale that reflected both the accuracy and sophistication of the strategy used: 1 = error in using the retrieval, fingers, or decomposition strategy; 2 = error in using a counting strategy, whether finger or verbal counting; 3 = correct use of the max- or sum-counting strategy; 4 = correct use of the min-counting strategy; 5 = correct use of retrieval-related strategies; and 6 = correct retrieval.

Number sets task

The stimuli for this task were arrays of objects (e.g., stars) in half-inch squares and Arabic numerals (18-pt font) in half-inch squares. Pairs and triplets of these stimuli were joined in domino-like rectangles, and the combinations of objects and numerals were varied. Five paired stimuli were presented in each line of a page except for the last two lines, which showed three triplets on each line. The target number (i.e., the target sum for each pair or triplet) was shown in large font at the top of each page. On each of the two pages for each target number (5 and 9), 18 items matched the target, 12 were larger than the target, and six were smaller than the target. Performance was consistent across target numbers and item content (i.e., object sets or Arabic numerals), so the investigator combined data across all items and calculated the overall frequency of hits and false alarms (Geary et al., 2018).

Number line estimation task

On each of the 24 trials, a 25 cm number line consisting of a blank line with two endpoints (0 and 100) was presented, along with a target number (e.g., 45) in a large font print above the line. The child's task was marking the line where the target number should lie. Accuracy was defined as the absolute difference between the child's mark position and the correct position of the number (e.g., for the number 45, marking the line at 35 or 55 would result in a score of 10). The overall score was the mean of these differences across the 24 trials.

Weekly Probes: Proximal Measures

The investigator administered the TEMI-AC as proximal measures (see Appendix E). The five alternate forms of TEMI-AC (A–E) were delivered in counterbalanced order on Fridays. TEMI-AC contains four 2-min fluency measures assessing magnitude comparisons (circling, from two numbers shown, the number that is lower or circling both numbers if they are equal), number sequences (writing the number that is missing from a three-number sequence), place value (writing how many hundreds, tens, and ones are pictorially depicted), and addition-subtraction combinations (solving basic addition and subtraction facts). The TEMI-ACs were aligned with the numerical skills and concepts taught in the intervention, which took approximately 10 minutes. The number and operation skills measured in the TEMI-ACs are essential for students to develop a foundation of number sense that is critical for later mathematics success (National Council of Teachers for Mathematics [NCTM], 2008). The raw scores of the four measures were summed, yielding a total score that could be used to monitor student progress. The TEMI-AC has five alternate forms; alternate-form reliability of the total score exceeds .80 across all forms.

Social Validity Measure

The investigator developed a social validity measure for students. A face rating scale along with items related to intervention content were utilized before and after the intervention to obtain data on students' perspectives.

Fidelity of Implementation

The investigator fully developed two fidelity measures and procedures for collecting fidelity data. The first fidelity form was administered to collect data on the degree to which all components of the lessons were conducted as written. The second fidelity form was fidelity of assessment that was measured through a checklist aligned to the scripted prompts and procedures of the screening and cognitive measures (see Appendix C). A trained research assistant with a background in special education recorded fidelity of intervention for 30% of intervention sessions and fidelity of assessment for 20% of assessment sessions.

PROCEDURES

After selecting the participants, the investigator followed the procedures: (a) screening, (b) pre-intervention, (c) baseline, (d) intervention, (e) maintenance, and (f) generalization. Table 3. 3 includes the phases and activities for this study.

Phase	Activities
Screening	• *Gate 1:* Result of universal screener at school (TEKS-aligned Pearson Education End of Year test) was used to identify students whose scores ranked below the proficiency level (70% accuracy on the test) • *Gate 2:* TEMA-3 (30 min) were administered to identify students whose scores ranked below the 30th percentile

Pre-intervention	• First-grade NSK measures (30 min) after screening
	  • Addition strategy task
	  • Number sets task
	  • Number line estimation task

Baseline	• TEMI-AC was administered twice a week (15 min), five to 10 data points across participants
Intervention	• 30 min intervention sessions, four times per week for six weeks TEMI-AC (15 min), twice per week (i.e., Tuesdays and Fridays)

Generalization	• TEMA-3 (30 min) after completion of the intervention
	• First-grade NSK measures (30 min):
	  • Addition strategy task
	  • Number sets task
	  • Number line estimation task

Maintenance	• TEMI-AC (30 min); two and four weeks after the last intervention sessions

Table 3. 3: Phase and Activities for the Study

Baseline Phase

During the baseline phase, the participants attended their regular mathematics instruction class schedules. The TEMI-AC was administered to the participants weekly at approximately the same time of the school day when future intervention sessions were implemented.

Intervention Phase

When a stable baseline (i.e., the data points are closer to the trend line; Horner et al., 2012; Kennedy, 2005) had been determined for the first student based on the total score of the TEMI-AC, the investigator administered the intervention sessions (30–35 minutes). Using multiple baseline procedures, intervention sessions began for each student in turn after he or she achieved a stable baseline on the total score of the TEMI-AC. Participants attended four intervention sessions (Monday through Thursday) and one review session (Friday) for approximately 30–35 minutes per week in the library for six weeks. Participants received TEMI-AC (Form A–E) as the progress monitoring measure twice per week (i.e., Tuesday and Friday). The investigator implemented the intervention sessions one-on-one in the school library on the same days and times each week (Monday through Thursday). Friday was reserved for review and progress monitoring. The intervention consisted of 24 lessons; students received four lessons (one lesson per session) during each week. Each intervention session included a warm-up activity and a lesson. Each lesson consisted of five major instructional components: (a) explicit instruction (i.e., modeled practice, guided practice, independent practice or practice testing), (b) multiple representations, (c) scaffolding, (d) feedback, and (e) students' verbalizations. Each lesson provided a script to use when implementing the intervention. Columns down the side of each lesson page provided student error-correction suggestions. Every student received a booklet each day that contained all modeled practice, guided practice, and independent practice sheets that students needed in the lesson.

The following mathematical concepts and skills, which were aligned with the first-grade Texas Essential Knowledge and Skills (TEKS), were taught to each student in the intervention phase: (a) addition/subtraction combinations, (b) number sequences, (c) magnitude comparisons, and (d) relationships of 10. Multiple learning principles were integrated into the design of intervention. Table 3. 4 displays the integration of learning principles into the early mathematics intervention. Each session included a warm-up and one lesson.

Learning Principles	Application
Elaborative Interrogation	During modeling and guided practice sections, the investigator encouraged learners to generate an explanation for an explicitly stated fact— examples include 'Why does it make sense that…?', 'Why is this true?', 'Why?', 'Why did you select that answer?' , 'What steps did you take to get that answer?', 'Why do you think it is true?', 'Why do you think so?'. Elaborative interrogation helps students to connect their prior knowledge with the new information, and to organize information and identify both similarities and differences between related entities (Dunlosky et al., 2013).
Interleaving practice	At the beginning of each intervention, the investigator presented a flashcard on the previously learned skills and asked the students to give a quick oral or written response (within five seconds). If students gave an incorrect answer to a flashcard, the investigator put the card in a pile for extra practice. The interleaving principle suggests that when practice problems are alternated, with a problem on one concept followed by a problem on another concept, students learn better than if problems are blocked, or grouped by concept (Rohrer, 2012). Spreading out learning opportunities leads to better long-term retention of information (Dunlosky et al., 2013).
Scaffolding	During modeling and guided practice, the investigator provided support to promote learning when concepts and skills are being first introduced to students. The investigator provided good models and used think-aloud to make thinking visible and easier for students to follow. Students had multiple opportunities to respond during the intervention. For students who are at risk for MD, the scaffolding through a short-term validated intervention creates a strong foundation to experience long-term success with their mathematics schooling (L. Fuchs, Fuchs, & Compton, 2012).
Multiple representation	The investigator selected, applied, and translated between concrete, symbolic, and abstract representations. The investigator encouraged students to draw connection between representations and notice contrasts between the relationships of two or more representations. Comparing multiple representations facilitates schema formation (Rittle-Johnson, Star, & Durkin). Also, linking two or three types of representations may support efficient learning (Goldstone & Son, 2005).

Table 3. 4: The Integration of Learning Principles into the Early Intervention

Work example	The investigators showed multiple worked out solutions to a problem and asked student the following question: 'suppose you are helping your teacher to grade a math test. These are students' responses; you need to decide which one is correct and which one is incorrect and why'. Researchers have found that having learners study examples of worked-out solutions to problems is more effective for learning than having them solve all of the problems themselves (Sweller & Cooper, 1985).
Error reflection	The investigator provided corrective affirmative feedback and prompted students to identify their errors and think about what features of the problem make the specific step taken incorrect. Studying errors provides exposure to multiple perspectives rather than just one's own perspective (Siegler and Chen, 2008). The investigator encouraged students to share their thinking aloud about their solution approaches and their mathematical understanding (Gersten et al., 2008).
Practice testing	At the end of each intervention session, students received a practice test sheet on the content that they were taught in that session. Practice testing may improve how well students mentally organize information and how well they process information which together can support better retention and test performance (Hunt, 1995, 2006)

Table 3.4 continued

Warm-up

Before the lesson each day, there was a warm-up activity that was designed as an interleaving practice. The investigator presented a flashcard on the previously learned skills (e.g., addition/subtraction combinations, relationships of 10) and asked the students to give a quick oral or written response (within five seconds). If students gave an incorrect answer to a flashcard, the investigator put the card in a pile for extra practice. After students went through all the cards, the investigator reviewed all the answers to cards in the extra-practice pile. There were four activities used by the investigator during the intervention: (a) naming numbers, (b) writing numbers from dictation, (c) orally answering previously learned addition and subtraction facts, and (d) orally answering previously learned relationships of 10 flashcards.

Lessons

Each day of intervention included one lesson (see Appendix D). These lessons were designed to teach the five skills (magnitude comparison, number sequence, relationships of ten and addition/subtraction combination) and to provide instruction to teach conceptual, procedural, and strategic knowledge. Each intervention lesson consisted of six major sections: lesson frame, lesson preview, modeled practice, guided practice, independent practice, a check for understanding and error correction.

The lesson frame (i.e., the first page of each lesson) was an overview of the lesson that included (a) the concept and skill being taught, (b) the name of the lesson, (c) the objective, (d) the instructional content (range of numbers), (e) the materials, (f) the vocabulary (i.e., the new mathematical vocabulary reviewed within the context of the lesson or taught separately using vocabulary strategies), and (g) the instructional time (total time, instruction time, and time for independent practice).

The preview section included the lesson goal (e.g., today we will learn strategies for + 2 facts), and questions that encouraged students to think aloud and share their experiences and prior knowledge about the concept of the day. In this section, the lesson's new vocabulary was reviewed. The goal of the preview section was to activate students' prior knowledge and to connect the new concepts to their daily lives.

The modeled practice section consisted of scaffolding, feedback, error corrections, and students' verbalization. In this section, the investigator taught the new concepts and procedures while engaging students during instruction. The purpose of modeling was to (a) model what students were to learn, (b) assist students as they worked through scaffolded instruction, (c) facilitate opportunities for students to engage in mathematical discourse, and (d) provide specific academic feedback (e.g., including error corrections and reasons for correct responses) to students during the mathematics activities. There was a suggested lesson script in each lesson that guided the investigator to model the concept and provide a clear explanation. The main part of modeling was scaffolding instruction that involved three components: (a) scaffolding was contingent on students' level of understanding, (b) there was a gradual withdrawal of scaffolding, and (c) there was a responsibility transfer from the investigator to the students. The investigator monitored the students' learning as they progressed and provided corrective informative feedback.

The guided practice section consisted of multiple opportunities to practice skills and concepts of the lesson (e.g., reading, writing, and making the numbers within a given number range). The goal of guided practice was to check for understanding, so the investigator asked exploratory questions and paid close attention to the students' responses. As required, the investigator provided error correction and used scaffolding to aid in student understanding.

The independent practice section was a form of practice testing that involved completing practice problems or questions on the concept of the day. The investigator asked the students to complete as many items as possible, as accurately as possible. After the independent practice, the investigator went through the items with students, telling them the correct answers. The students were asked to put a check mark by correct answers and correct any errors. At the end, students recorded their scores as the number correct / total number possible.

The investigator promoted students' verbalization by asking questions that could be answered using information from the lesson. This form of the explanatory questions included but was not limited to "Why is this true?", "Why does it make sense that…?", and simply "Why?". The investigators used multiple representations (i.e., concrete, semi-concrete, and abstract) as needed during the lesson. These representations depicted concepts in different ways to help students develop conceptual knowledge.

Materials

There were four types of materials: (a) hands-on materials that were used with many lessons, such as flashcards, relationships of 10 cards, ten-frames, math manipulatives (e.g., connecting cubes, two-color counters, base ten blocks) and pictures that represented the concrete objects previously used; (b) templates or charts (e.g., hundred charts, ten-frames); (c) worksheets (i.e., modeled practice, guided practice and independent practice); and (d) managing materials that the investigator used to keep material organized and easily accessible during instruction. A storage container with materials and lessons for each day was used to facilitate smooth instruction. The investigator provided the following materials: base ten blocks (flats, rods, units), connecting cubes, counters (e.g., paper clips, small stones, pennies), timers, and wipe boards as well as dry-erase markers.

Treatment Integrity

The intervention fidelity was evaluated through four components, the first being the tutors' ability to implement instructional procedures during the intervention phase. The second evaluating component was the tutors' effectiveness in using explicit instruction (e.g., increasing scores). The third component was the tutor's ability to promote students' verbalization by using high cognitive questions (e.g., Why? How?). The last component was the quality of the intervention (e.g., making the students feel valued and welcome; being responsible for the student's behaviors). Each component indicator was rated using a 3-point scale from poor to excellent with 1 being *poor* and 3 being *excellent*. To assess the fidelity of implementation, two trained two doctoral students observed 25% of intervention sessions for each participant (total of 12 lessons). The fidelity checklist included three qualitative questions at the end (i.e., strengths observed, suggestions for improvement, the overall impression of teaching effectiveness) that the observer discussed with the investigator after each observation. Across all observations, the highest rating (3) was given on the level of *interventionist competence*. On a few occasions, the interventionists received scores of 2 (Good) on promoting self-explanation and verbalization (e.g., prompting students to answer *why?* questions: *why does it make sense that…, why is this true*, and simply *why?*). The fidelity of intervention overall ranged from

81% to 100% throughout the intervention, with an average score of 92.5 (SD = 6.53). The investigator and a trained doctoral student double scored 100% of progress monitoring data. The interscorer agreement was calculated by adding the agreements and dividing by the number of agreements and disagreements. The mean interscorer agreement was 98% for all progress monitoring forms across all students and phases.

Generalization Phase

After the last intervention session, a trained research assistant with a background in special education administered a post-intervention, distal measure (i.e., TEMA-3) to assess the generalization of early numeracy knowledge and skills.

Maintenance Phase

This phase took place after the conclusion of the last intervention session for each student. No further intervention sessions took place between the end of the intervention phase and the administration of maintenance measures. To assess maintenance, the TEMI-AC was administered to each of the participants during the typically scheduled intervention time two and four weeks after the final intervention session.

DATA ANALYSIS

The investigator graphed and analyzed the data for the TEMI-AC score on a weekly basis to determine baseline stability and intervention progress. The investigator visually analyzed level, trend, variability, immediacy of the effect, and consistency of data patterns on an on-going basis as the study was executed. Additionally, statistical analysis was conducted using the nonoverlap of all pairs (NAP) approach as a non-parametric ES. Finally, descriptive statistics were utilized for the early mathematics pre- and post-tests (i.e., TEMA-3, addition strategy task, number sets task, and number line estimation task), intervention fidelity, and social validity. The analytic strategies are explained further in the following sections.

Visual Analysis of Proximal Measures

Visual analysis was completed using the guidelines and standards recommended by the What Works Clearinghouse (WWC; Kratochwill et al., 2010) to evaluate the effectiveness of the explicit systematic early mathematics intervention on the mathematical performance of students with MD (Kennedy, 2005). The analytical procedure required the assessment of participants' responses to dependent variables through graphical data across phases (i.e., baseline, intervention, maintenance). Visual data in graphical form was evaluated to determine if a functional relation between the independent variable (the early mathematics intervention) and participants' mathematical outcomes was present (Kennedy, 2005). The WWC rules for conducting visual analysis involve six features and four steps that are described below (Parsonson & Baer, 1986).

Six features of visual analysis

To examine within- and between-phase data patterns, six features were used: (a) level, (b) trend, (c) variability, (d) immediacy of the effect, (e) overlap, and (f) consistency of data in similar phases (WWC; Kratochwill et al., 2010). These six features were assessed to determine whether the results from the current study demonstrate a functional relation. The six visual analysis features were utilized collectively to compare the observed and projected patterns for each phase with the actual pattern observed after manipulation of the independent variable (the early mathematics intervention). This comparison of observed and projected patterns was conducted across all phases of the design (e.g., baseline to intervention, intervention to baseline, intervention to intervention; WWC; Kratochwill et al., 2010). The level was defined as the average of the scores across a given phase. The investigator determined the trend by visual analysis of the data through the phases (Kratochwill et al., 2010). Variability within the phase was demonstrated by the standard deviation of the data in relation to the trend line (Kratochwill et al., 2010). The immediacy of effect was determined by comparing the extent to which the level, trend, and variability of the last three data points in one phase were distinguishably different from the first three data points in the next (Kratochwill et al., 2010). Overlap of data refers to the percentage of data from one phase that overlaps with the data in the previous phase (Kratochwill et al., 2010). The consistency of data was identified by looking at data from all phases within the same condition (e.g., all baseline phases, all intervention phases) and identifying if there was consistency in the data patterns from phases with the same conditions (Kratochwill et al., 2010).

Four steps of visual analysis

The procedure for conducting visual analysis involved four steps that the WWC recommended (Kratochwill et al., 2010). The first step in the visual analysis was to examine whether the data in the baseline phase documented that (a) the proposed problem was demonstrated (e.g., having difficulties in mathematics) and (b) the data provided sufficient demonstration of a clearly defined (i.e., predictable) baseline pattern of responding that could be used to assess the effects of the intervention. The two purposes of a baseline were to (a) document a pattern of behavior in need of change and (b) document a pattern that had a sufficiently consistent level and variability, with little or no trend, to allow comparison with a new pattern following intervention. The second step in the visual analysis process was to assess the level, trend, and variability of the data within each phase and to compare the observed pattern of data in each phase with the pattern of data in the next phases. In the third step, the information collected through examination of level, trend, and variability was supplemented by comparing the overlap, immediacy of the effect, and consistency of patterns in similar phases. The final step of the visual analysis process included combining the information from each of the phase comparisons to determine whether all the data in the design (data across all phases) met the standard for the demonstration of experimental control.

After analyzing and comparing phases in the six features and four steps given by the WWC (Kratochwill, et al., 2010), a treatment effect was considered present if there was a change in level between the baseline and intervention phases of the study. In sum, students' data points from baseline through intervention phases, as well as within the maintenance phase (i.e., two and four weeks after the last intervention session) were assessed. Visually evaluating the six characteristics of the graphical data and the demonstration of prediction, verification, and replication in the data allowed the investigator to identify the effects of the independent variable (the early mathematics intervention) on dependent variables (Kratochwill et al., 2010).

Proximal Effect Sizes

The NAP approach was used to assess the effectiveness of the intervention. The NAP approach has been demonstrated and field-tested in over 200 published single-subject studies (Parker & Vannest, 2009). NAP has also been included in recently proposed standards for evaluating single-subject design research (Horner et al., 2005). The investigator examined the extent to which data in the baseline versus intervention phases did overlap as an accepted indicator of the amount of performance change.

NAP was used not as a test on means or medians but rather on location of the entire score distribution and was not limited to a particular hypothesized distribution shape (Parker & Vannest, 2009). There are two procedural options for NAP hand calculation. One may begin by counting all nonoverlapping pairs, or by counting all overlapping pairs and subtracting from the total possible pairs to obtain the nonoverlap count. The total possible pairs (total N) is the number of data-points in phase A times phase B ($N_A \times N_B$). For most datasets, it is faster to begin counting only overlap, then subtract from the total possible pairs (Parker & Vannest, 2009).

Generalization

The generalization of NSK was assessed through a post-intervention administration of the three measures (addition strategy task, number sets task, and number line estimation task). The descriptive analyses (i.e., means and standard deviations) were used to compare participants' results in the pre-test to the results in the post-test. Also, the generalization of students' early mathematics knowledge and skills were evaluated. The TEMA-3 was administered to the participants two weeks after attending the last intervention session. The scores obtained on the TEMA-3 were compared to the pre-intervention scores obtained on the same measure. To analyze the data from the social validity questionnaire, the investigator computed mean scores for each of the closed-ended questions of the questionnaire across all participants. Then the mean score for each question was used to determine the perspectives of participants toward each question.

Chapter 4: Results

Mathematical knowledge at school entry is the strongest predictor of future academic success in mathematics but also in other academic domains (Duncan et al., 2007). Children who leave kindergarten below the 10th percentile in mathematics have a 70% chance of remaining at this level 5 years later and would be classified as having MD (Morgan et al., 2009); thus, early mathematics interventions for such at-risk students are vital for all aspects of academics. The purpose of this study was to investigate the effect of an early mathematics intervention delivered four times weekly for 35 minutes in twenty-four sessions, one on one, through a mathematics instructional approach using multiple learning techniques. The study measured the effectiveness of the early mathematics intervention by using a pre/post- test comparison in the areas of early mathematics skills. It also progress-monitored the subject in magnitude comparison, number sequence, place value, and addition and substation combination. This chapter includes the results of the visual, descriptive, and statistical analyses to answer the following research questions.

6. Does the early mathematics intervention result in improved performance on the TEMI-AC, which is a weekly proximal measure of mathematics?
7. Does the early mathematics intervention result in improved performance on the NSK task, which is less proximal to the intervention?
8. Does the early mathematics intervention result in improved performance on the TEMA, which is a distal mathematics measure?

9. Are the effects of the intervention maintained two- and four-weeks post-intervention based on performance on the TEMI-AC?
10. What are the students' perspectives on this early mathematics intervention?

The chapter will begin with a discussion of intervention integrity, followed by the research questions and their related results. Proximal data related to students' weekly total scores (i.e., TEMI-AC) is presented in Figure 4. 1. Finally, a brief overview of the results will be provided at the end of the chapter.

RESEARCH QUESTION ONE

Research question one examined the effects of early mathematics intervention on the mathematics performance of first-grade students at risk for mathematics difficulties (MD). The TEMI-AC (University of Texas System/Texas Education Agency, 2009) was administered to assess the early mathematics concepts and skills of students at risk for MD. The investigator used the total score of TEMI- AC to evaluate students' progress twice weekly.

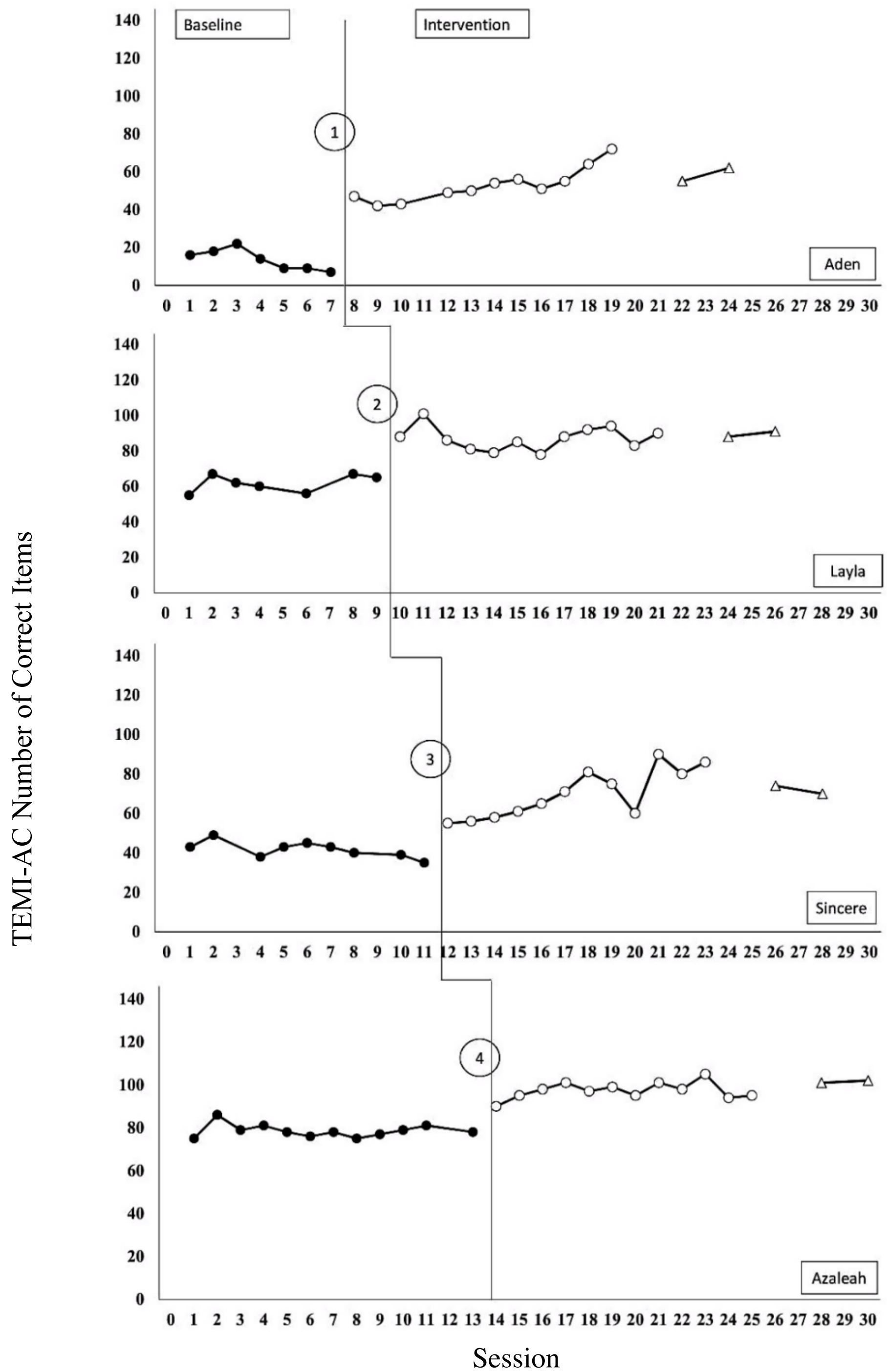


Figure 4. 1: Multiple baseline design documenting four demonstrations of the effect of early mathematics intervention.

TEMI-AC = Texas Early Mathematics Indicators–Aim Checks

Visual Analysis

As recommended by What Works Clearinghouse (WWW, 2014), six features were used to examine within- and between-phase data patterns to assess the effects of explicit strategic early mathematics intervention within single case design: (a) level, (b) trend, (c) variability, (d) immediacy of the effect, (e) overlap, and (f) consistency of data in similar phases (Fisher, Kelley, & Lomas, 2006; Kennedy, 2005; Morgan et al., 2009; Parsonson & Baer, 1978). These six features were analyzed to determine if a causal relationship existed between the early explicit strategic mathematics intervention (i.e., independent variable) and the early numeracy knowledge and skills of students at risk for MD as exhibited through their performance (i.e., total score) on TEMI-AC (i.e., dependent variable). Table 4. 1 indicates the level and trend data for the participants. Table 4. 2 shows variability (i.e., the fluctuation of data around the mean score indicated by standard deviation and range of data), the immediacy of effect and overlap of data between the baseline and intervention phase for participants.

Students	Level			Trend		
	Baseline	Intervention	Maintenance	Baseline	Intervention	Maintenance
Aaden	13.57	53	59	-2.07	2.1	3.5
Layla	61.72	87.08	90	0.63	-0.03	1.5
Sincere	41.66	69.83	72	-0.83	2.85	-2
Azaleah	78.58	97.33	101.5	-0.13	0.31	0.5
Mean	48.88	76.81	80.63	-0.6	1.31	0.87
(SD)	(27.96)	(19.51)	(18.85)	(1.14)	(1.38)	(2.29)

Table 4. 1: Level and Trend Data for Participants

Students	Variability standard deviation (Range)		Immediacy of effect (%)	Overlap
	Baseline	Intervention		
Aaden	5.50 (7-22)	8.84 (42-72)	35.66	No
Layla	4.96 (55-67)	6.63 (78-101)	29	No
Sincere	4.15 (35-49)	12.33 (55-90)	18.33	No
Azaleah	3.06 (75-86)	3.94 (90-105)	15	No
Mean (SD)	4.42 (1.06)	7.94 (3.55)	24.50 (9.54)	

Table 4. 2: Variability, Immediacy of Effect, Overlap Data for Participants

Experimental control involves replication of the intervention in the experiment. In multiple baseline design, the replication is addressed with the staggered introduction of the independent variable across different points in time (WWC, 2010). This replication of effect is important for controlling threats to internal validity. In this study, the experimental control was established by the arrangement of conditions and manipulation of the independent variable across four different points in time (Bloom, Fischer, & Orme, 2009). Figure 4. 1 showed four demonstrations of predicted effect at four different points in time (Fisher, Piazza, & Roane, 2011; Horner et al., 2005; Kratochwill et al., 2010). To assess experimental control in the study, first, the data from adjacent phases were compared, and then the data patterns from all phases in the study were integrated. For example, to evaluate the effect across baseline and intervention phases (e.g., determine if the introduction of an intervention produced a predicted change in the early numeracy knowledge and skills), data from the second phase were compared initially with the data from the first phase and then with the “projected results” (e.g., extension of the data pattern from the first phase into the second phase). In each case, data in the second phase were examined and compared (a) with the actual data from the first phase and (b) with the expected, or projected, data pattern (with confidence intervals) obtained by extending data from the first phase into the second phase (the shaded areas in Figure 4. 2). Visual analysis of data involves the simultaneous assessment of the level, trend, and variability of the data within and across adjacent phases. When data from two adjacent phases were compared, the rules of visual analysis also included assessment of immediacy of effect, the level of overlap, and the consistency of data patterns in similar phases (Parsonson & Baer, 1978). The role of each of these variables in the visual analysis is described in the following section.

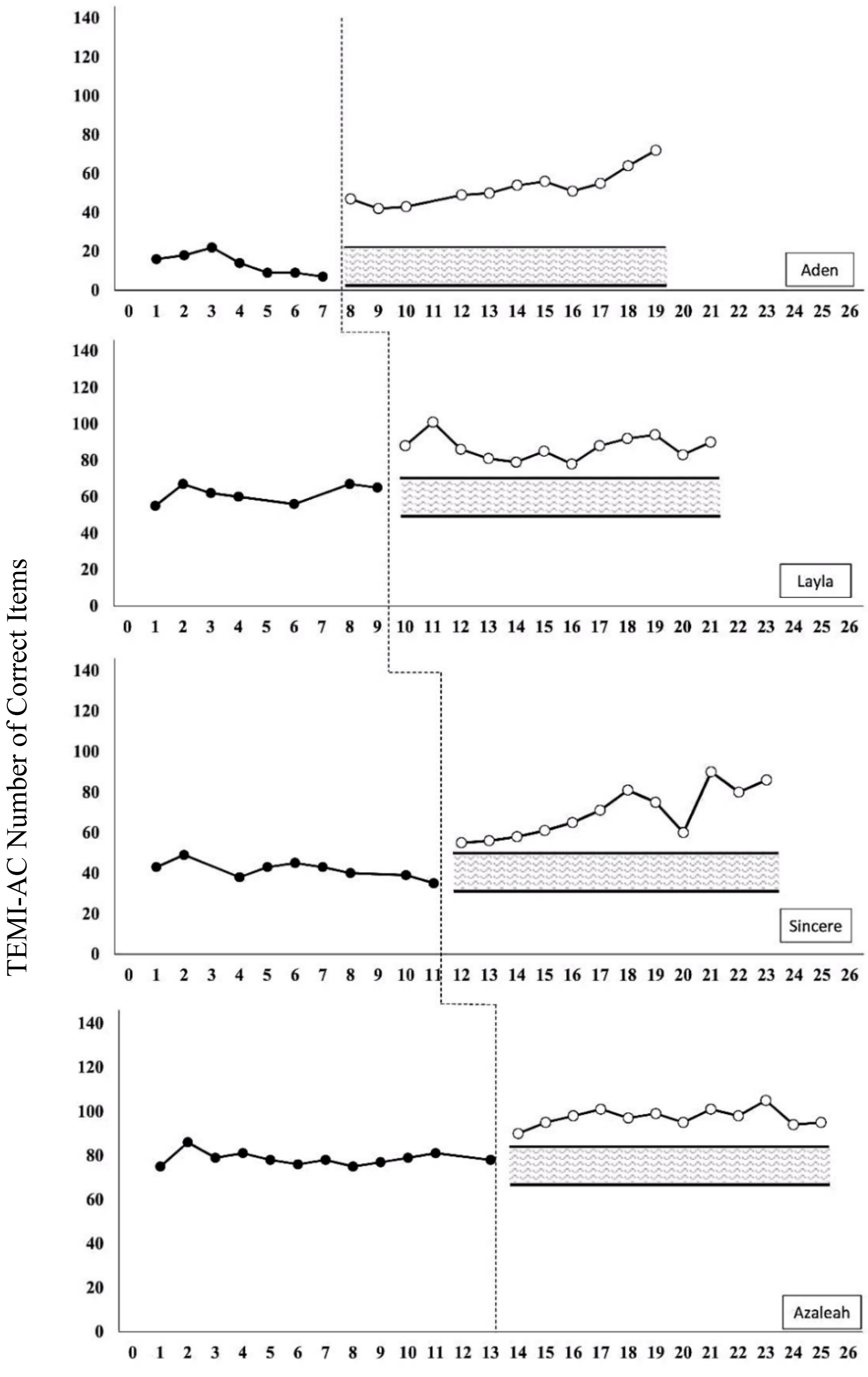


Figure 4. 2: Baseline and intervention phases

Shaded area indicates the expected data (with confidence intervals) in the intervention phase based on data in the baseline phase.

Baseline

Active documentation of performance under baseline is the center feature of single-case design research. In this study, the baseline phase included between 7 to 13 data points. The baseline data showed (a) the current pattern of responding and (b) a confident prediction of the pattern of future responding. The investigator collected the baseline data individually in a quiet space in the school library, where later intervention sessions and progress monitoring happened. All the phases happened in the same place under the same condition to assure that only the independent variable was altered at the point of intervention. All other baseline variables were held constant so that the independent variable was likely to be responsible for the change in the dependent variable (the early numeracy knowledge and skills).

Figure 4. 3 displays students' twice-a-week TEMI-AC total scores during each phase. Figure 4. 4 and Figure 4. 5 show level and trend data, respectively. Table 4. 6 and Table 4. 7 show variability, the immediacy of effect, and overlap data.

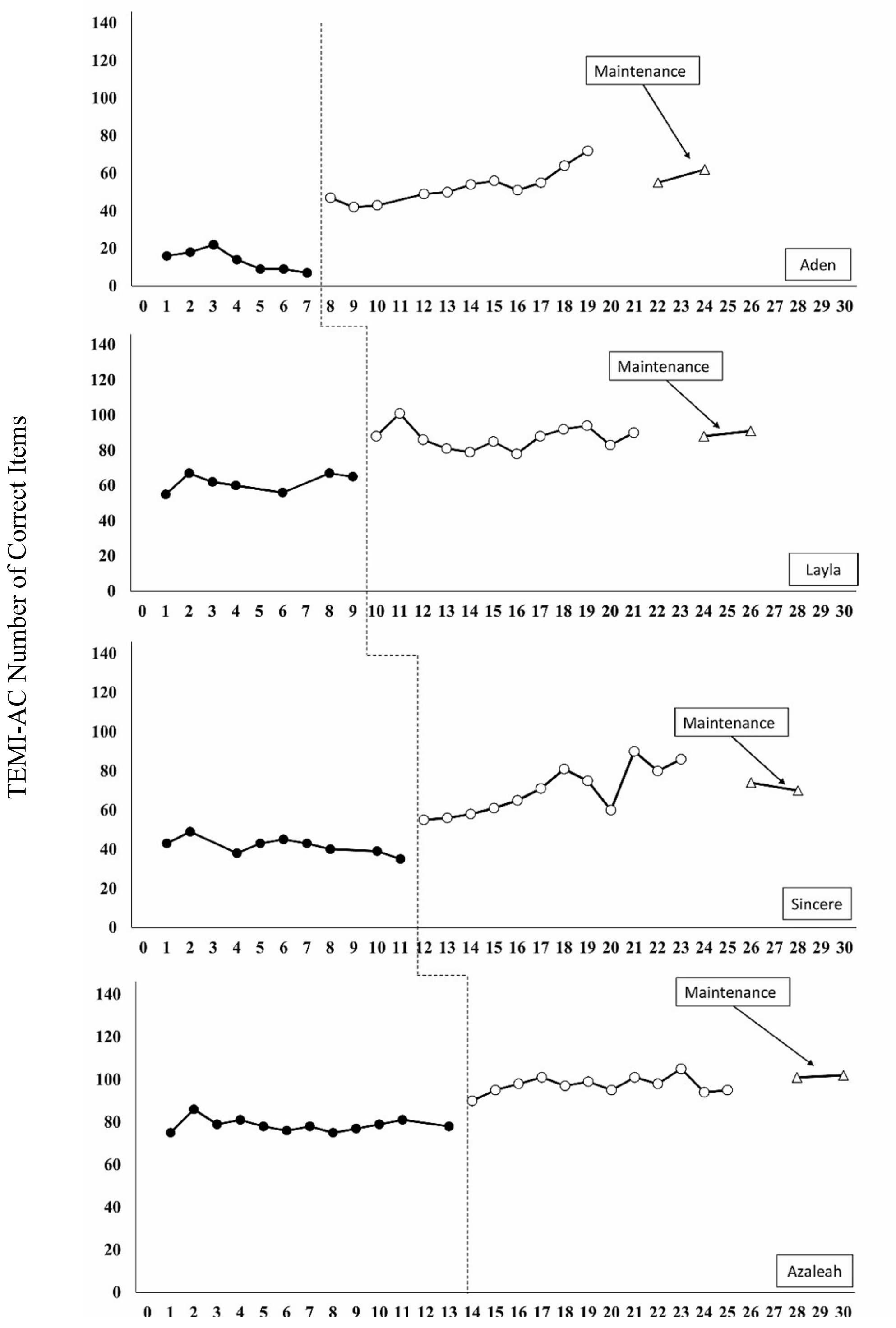


Figure 4. 3: Participants' TEMI-AC total scores by session

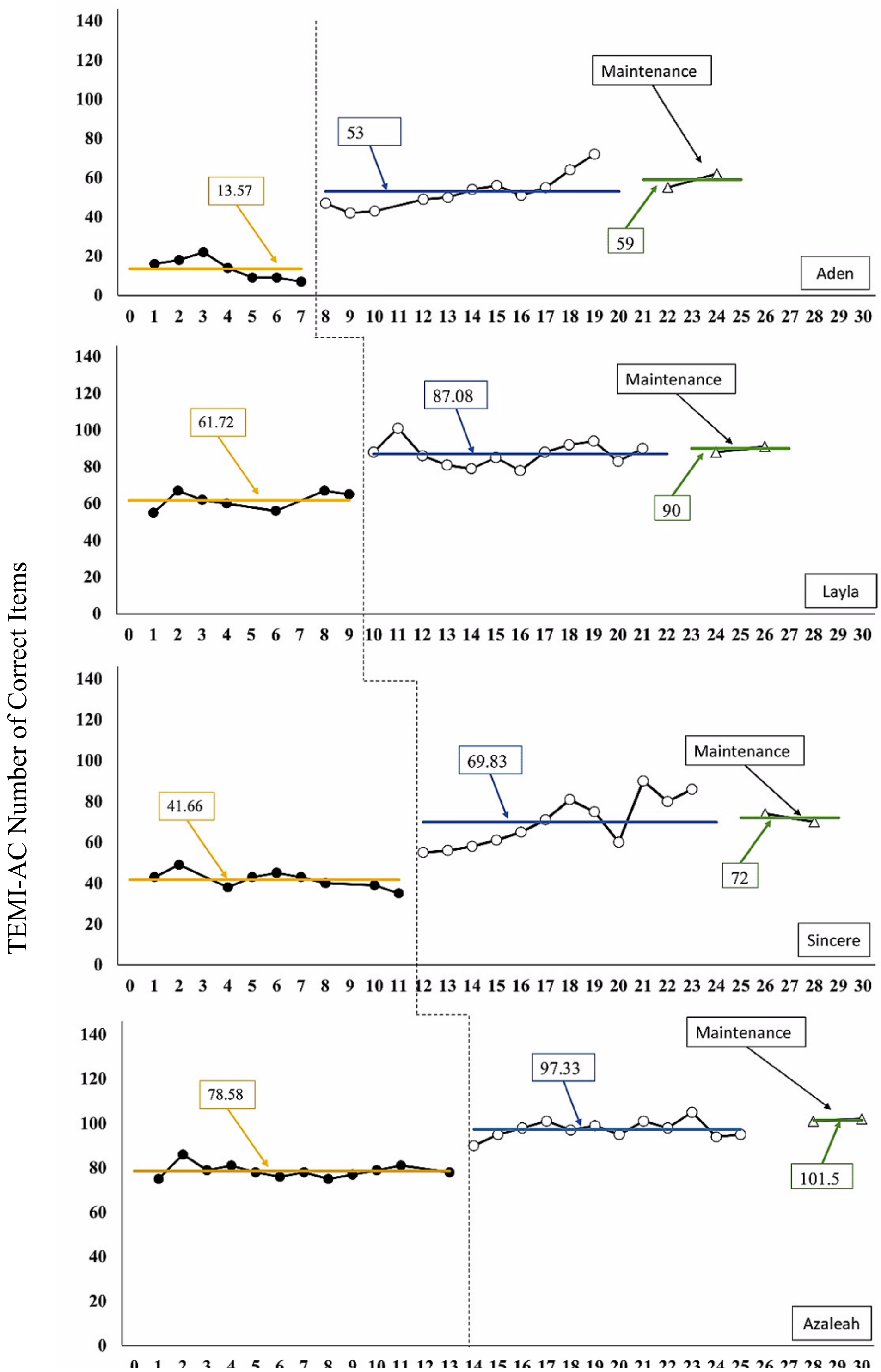


Figure 4. 4: Level for students' TEMI-AC total scores

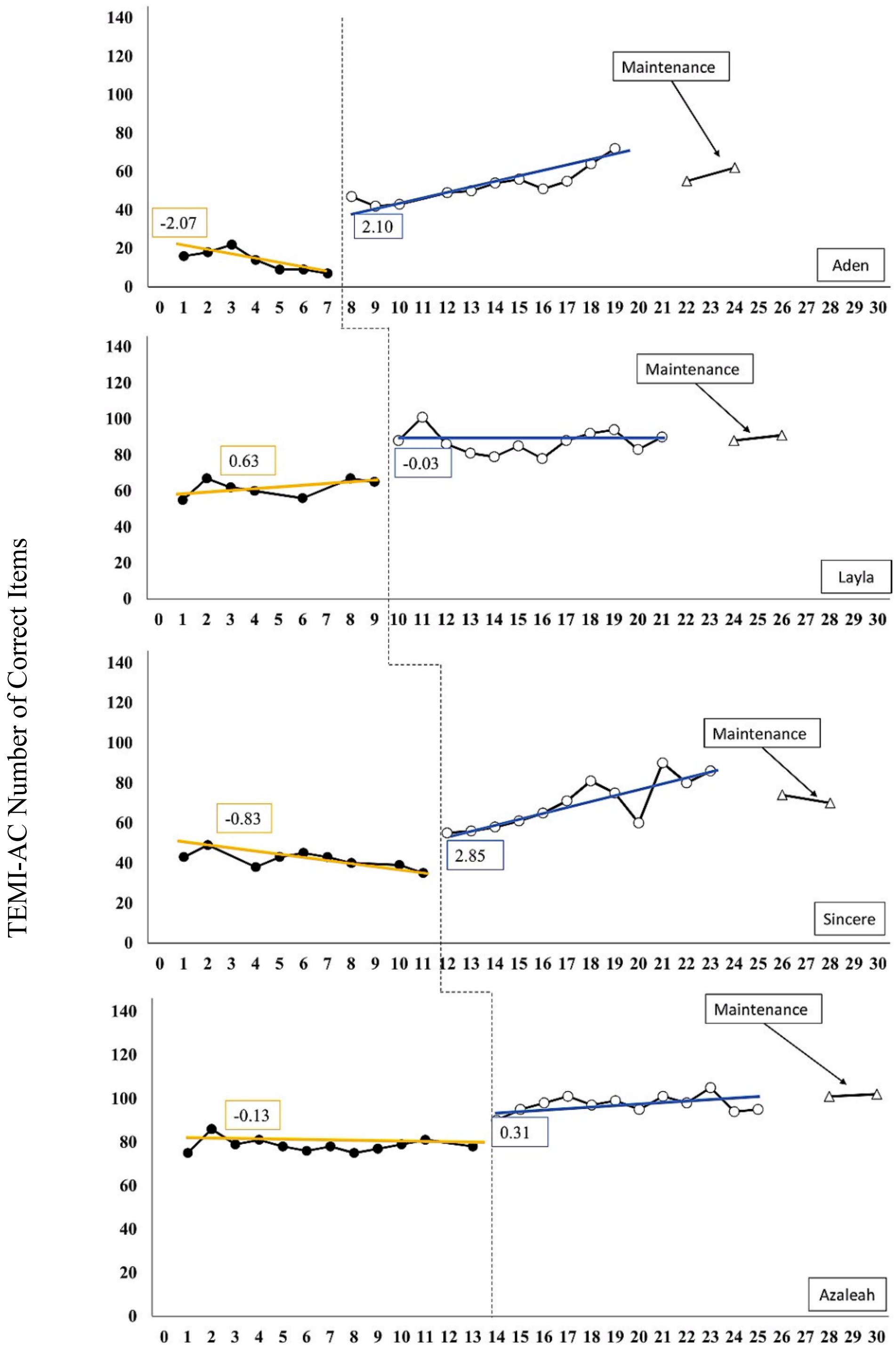


Figure 4. 5: The trend for students' twice-a-week TEMI-AC total scores

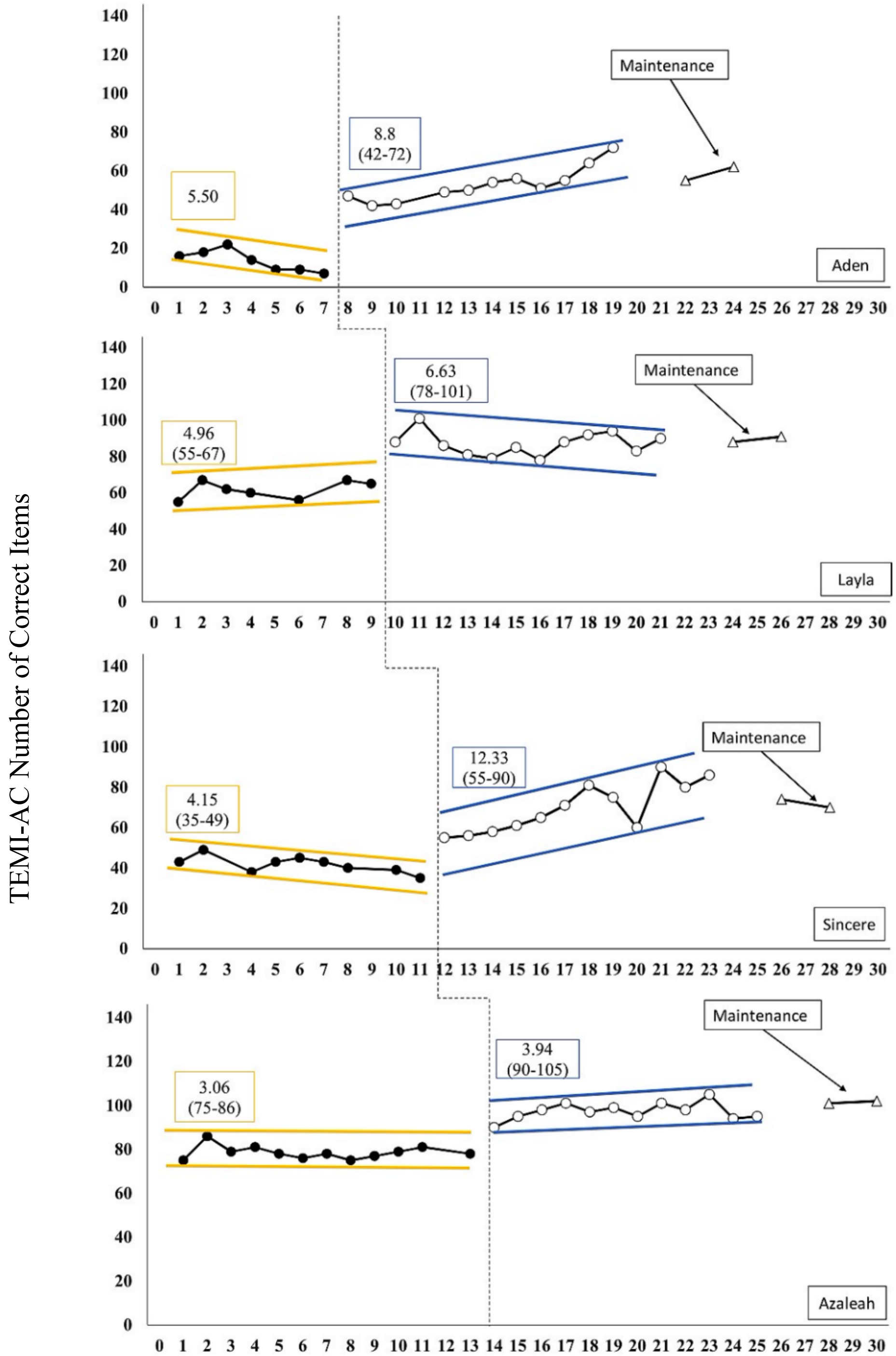


Figure 4. 6: Students' variability and immediacy of effect

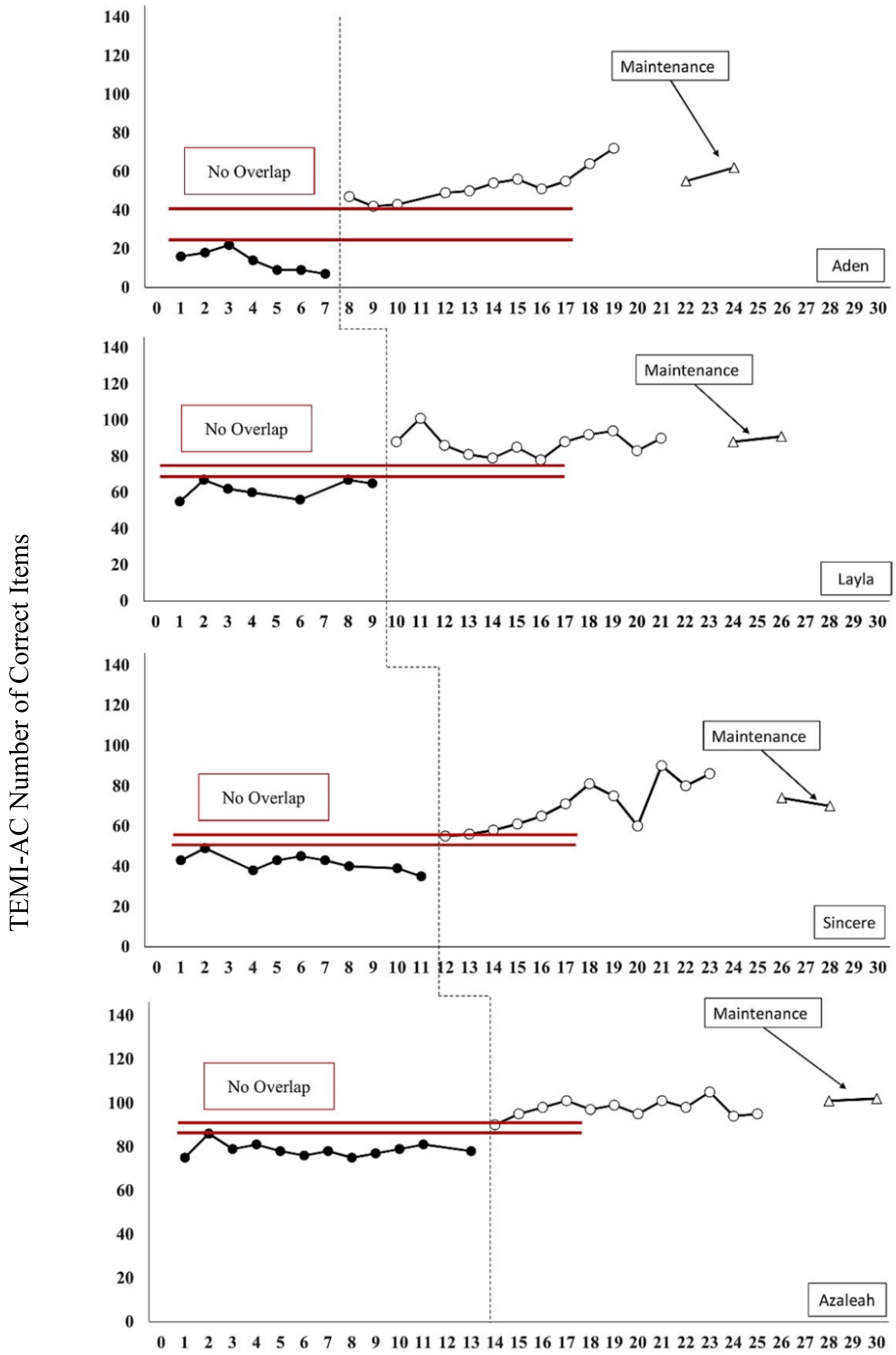


Figure 4. 7: Students' overlap data across baseline and intervention phases

Aaden

The first dimension used in the visual analysis is the level of the data that refers to the average of the data within a condition. There were 7 data points in Aaden's baseline, and he had the lowest level baseline data among all participants. The TEMI-AC total score level during his baseline phase was 13.57 and then increased considerably to 53 during the intervention phase (see Figure 4. 4). During the baseline, his scores were consistently low and stable (M = 13.57, range 7 to 22). After implementing the intervention (i.e., the explicit, systematic strategic early mathematics intervention), his TEMI-AC total scores immediately increased (M = 53, range 42 to 72). The level of maintenance data was 59 and larger than the level during the baseline and intervention phases. The level data show that the TEMI-AC total scores increased across the study and were maintained two and four weeks after the intervention was completed.

The second element of visual analysis in single-case design graphs is the trend of the data, which refers to the best-fit straight line that can be placed over the data within each phase (Horner et al., 2005). The two trend components that must be evaluated concurrently include slope and magnitude. Within each phase, the slope can be positive (upward) or negative (downward). Aaden's data points were negative during the baseline (-2.07) and positive during the intervention phases (2.10), meaning an increase in value between phases and within the intervention phase (see Figure 4. 5). Magnitude is the second component of the trend, which refers to the size or extent of the slope and can be qualitatively estimated as high, medium, and low (WWC, 2010). Aaden's TEMI-AC total score within the intervention phase showed a medium magnitude slope with a gradually increasing pattern in the data. During the intervention phase, Aaden showed an upward trend in TEMI-AC total scores, which was the largest immediacy of effect among all of the participants. There was no overlap of data between the baseline and intervention phase for Aaden (see Figure 4. 7).

Variability is the next element used to inspect graphs visually (see Figure 4. 6). Variability is defined as the degree to which each data point deviates from the overall trend. Variability can also be defined as the degree to which the data points were dispersed relative to the best-fit straight line (Kratochwill et al., 2010). The terminology used to define the variability of the graph is typically qualitative, including high, medium, or low. For Aaden, variability during baseline was medium (SD = 5.50; range = 7-22) and continued to increase during intervention (SD = 8.8; range = 42-72).

Layla

In terms of level, Layla's TEMI-AC total score level from baseline to intervention phase increased from 61.72 to 87.08 (See Figure 4. 4). There were 7 data points in Layla's baseline, and her first TEMI-AC total score during the intervention phase was above all her data points during the baseline phase. Figure 4. 4 showed that Layla's total score level increased by 25.36 from baseline to intervention phases. Layla showed the most stable intervention among all participants (M = 87.08, range 78 to 101). As presented in Figure 4. 4, Layla's data points in baseline were closer to the trend line, which showed the stability of the data points in the baseline phase. During baseline, her trend of TEMI-AC total scores was upward (0.63) and was nearly stable during the intervention phase (-0.03). Her maintenance data level (90) was above the baseline level (61.72) and was close to the intervention phase level (87.08).

The variability of the data points during the baseline phase was medium (SD = 4.96; range = 55-67). The level of variability increased during the intervention phase scores (SD = 6.63; range = 78- 101). No overlap data was observed during the intervention and baseline phases (see Figure 4. 7).

Sincere

Sincere's baseline level was stable with a downward trend (-0.83; see Figure 4. 5) and a level of 41.66 (see Figure 4. 4 and Figure 4. 5). The last three data points before intervention showed a downward trend (See Figure 4. 5). During the intervention, his level of TEMI-AC total score increased to 69.83 and showed an upward trend (2.85). Variability during baseline was low (SD = 4.15; range = 35-49) and increased during the intervention phase (SD = 12.33; range = 55-90). He had the most variation during the intervention phase compared to other participants. The maintenance data level was 72, which is quite larger than the intervention level and higher than the baseline level (41.66). Both the 2-week and 4-week maintenance data points exceeded all TEMI-AC total scores within the baseline phases. There were no overlap data points across the phases (see Figure 4. 7).

Azaleah

Azaleah's twice-a-week TEMI-AC total score level during baseline phase was 78.58 and then increased to 97.33 during the intervention (see Figure 4. 4). Azaleah's data level across all phases (baseline, intervention, maintenance) was the highest among all participants (see Figure 4. 4). After implementing the intervention, her TEMI-AC total scores increased and continued at relatively high and increasing levels for the rest of the intervention phase (M = 97.33, range 90 to 105). Azaleah's data level in the maintenance phase (101.5) was quite above the intervention phase level (97.33) and higher than baseline level (78.58), showing her upward trend in performance on TEMI-AC total scores two and four weeks after the completion of the intervention. Evaluating the consistency of data patterns showed Azaleah had the least amount of variation in baseline data (SD = 3.96; range = 72-86) and the intervention phase (SD = 3.94; range = 90-105). Azalea showed the most stable baseline of all participants, with a very small downward directional trend in TEMI-AC total scores (-0.13). During the intervention phase, Azaleah showed an upward trend in TEMI-AC total scores (0.31).

Summary of Research Question one

After introducing the intervention, all four participants showed improvement in their mathematical performance using the TEMI-AC total scores. The level data shows that the TEMI-AC total scores improved across the study and were maintained two and four weeks after the intervention was completed. Across all participants, the average level in baseline was 48.88 (SD = 27-96), which increased considerably during the intervention phase (M = 76.81, SD = 19.51) and maintenance phase (M = 80.63, SD = 19.51). The trend analysis indicated that three participants showed a downward trend during the baseline (M = -0.6, SD = 1.14) and an upward trend during the intervention (M = 1.31, SD = 1.38). The mean variability standard deviation across participants' TEMI-AC bi-weekly total scores was 4.42 (SD = 1.06) during baseline and 7.94 (SD = 3.55) during the intervention. The average immediacy of effect across all participants was 24.50 (SD= 9.54, rage= 15- 35.66), meaning after the implementation of the intervention, all participants showed high immediacy of effect from baseline to intervention phase (see Table 4. 1 and Table 4. 2). Furthermore, performance on TEMI-AC showed maintenance of scores at 2- and 4-weeks after the intervention across the participants. Based on the visual analysis findings, a causal relationship was demonstrated between implementing the explicit, systematic strategic early mathematics intervention and the mathematical performance of first-grade students at risk for MD.

Effect Sizes (TEMI-AC)

The Non-overlap of All Pairs (NAP) approach (Parker & Vannest, 2009; Horner et al., 2005) was computed as another method to assess the effectiveness of the intervention. Using the NAP approach, the investigator examined the extent to which the TEMI-AC data in the baseline versus intervention and maintenance phases did overlap (see Table 4. 3). NAP results were analyzed according to the following scale: 90-100% = large or highly effective, 70%-90% = moderately effective, and < 70% = small or questionable effectiveness (Ma, 2006).

Non-overlap of All Pairs (%)	Aaden	Layla	Sincere	Azaleah
Between baseline and intervention phases	100	100	100	100
Between intervention and maintenance phases	100	100	100	100

Table 4. 3: Non-overlap of All Pairs Across all Phases

For all participants, the average possible pairs between baseline and intervention phases were 103.25 data points (rage 77 - 144), and NAP was 100% which showed that from baseline to the intervention phase, data demonstrated a strong improvement (Parker & Vannest, 2009). There was also no overlapped data point between phases across participants (NAP = 100%). The 100% NAP value demonstrates a large effect (Parker & Vannest, 2009) of explicit, systematic strategic early mathematic intervention across participants, verifying a causal relationship between the introduction of the intervention and changes in participants' mathematical performance on TEMI-AC (Horner et al., 2005) at four different time points. The effect of the explicit, systematic strategic early mathematic intervention on TEMI-AC scores can be interpreted as being highly effective during both the intervention and the maintenance phases compared to baseline (Ma, 2006). NAP (100%) during the maintenance phase demonstrated large, long-lasting effects on TEMI-AC scores two and four weeks after the last intervention session.

Research Question Two

Research question two assessed the effects of explicit, systematic strategic early mathematics intervention on a less proximal measure, Number-System Knowledge (NSK) for students with MD. NSK consists of three sub-tests, including addition strategy choices, number sets, and number line estimation.

Addition Strategy Task

The participants were asked to solve fourteen simple and six complex addition problems that were presented on flash cards one at a time. The investigator asked the students to solve each problem as quickly as possible and to explain how they solved it. According to the participant's explanation and the investigator observation, each trial fell into one of the following categories: (a) counting fingers, (b) verbal counting, (c) retrieval, (d) decomposition, (e) finger (the child looked at his/her fingers without counting them), (f) other/mix, (g) no data. After collecting the data, the problems were coded on a 6-point scale that reflected both accuracy and sophistication of the strategy used: 1 = error in using the retrieval, fingers, or decomposition strategy; 2 = error in using a counting strategy, whether finger or verbal counting; 3 = correct use of the max or sum counting strategy; 4 = correct use of the min counting strategy; 5 = correct use of retrieval-related strategies (fingers and decomposition); 6 = correct retrieval. The results of pre/post-test are shown in Table 4. 4. The results demonstrated that participants' performance in the addition-strategy task improved significantly after the intervention (g = 1.61). Table 4. 5 shows the percentage of using addition-strategy task strategies before and after the intervention.

Participants	Addition-strategy task Pre	Addition-strategy task Post
Aaden	29	39
Layla	47	63
Sincere	41	60
Azaleah	31	49
Mean (SD)	37 (8.49)	52.8 (11)
Effect size (Hedges g)	1.61	

Table 4. 4: Participants' Pre-/Post-Intervention Total Scores in the Addition-Strategy Task

Accuracy and sophistication of the strategy	Aaden		Layla		Sincere		Azaleah		Overall percentage	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Error in using the retrieval, fingers, or decomposition strategy	75	40	50	25	45	35	65	30	58.75	32.5
Error in using a counting strategy, whether finger or verbal counting	5	35	20	20	30	15	15	15	17.5	21.25
Correct use of the max or sum counting strategy	20	15	5	20	10	5	20	45	13.75	21.25
Correct use of the min counting strategy	0	10	10	10	10	25	0	5	5	12.50
Correct retrieval	0	0	15	25	5	20	0	5	5	12.50

Table 4. 5: Percentage of Accuracy and Sophistication of the Strategy Used Before and After the Intervention

Number Sets Task

The participant was asked to move across each line of the page from left to right without skipping any items, to “circle any groups that can be put together to make the top number,” and to “work as fast as you can without making many mistakes.” There was a time limit of 60 s per page for target 5 and a time limit of 90 s per page for target 9 (Geary, 2018). The overall frequency of hits and false alarms were calculated for each participant. The signal detection measure, d-prime, was calculated for each participant by subtracting the standardized number of false alarms from the standardized number of hits. The signal detection d-prime measure is the difference between the means of standardized hits and standardized false alarms (Geary, Bailey, and Hoard, 2009).

The results are presented in Table 4. 6. The results indicated that participants’ performance in number sets task improved significantly in the post-test (Hedges' g = 1.86).

Participants	Signal detection measure d′ in pre-test	Signal detection measure d′ in post-test
Aaden	3	4.8
Layla	5.4	6.4
Sincere	3.4	7
Azaleah	4.6	5.8
Mean (SD)	4.1 (1.1)	6 (0.93)
Hedges' g	1.86	

Table 4. 6: Participants’ Pre-/Post-Intervention Scores in Number-Sets Task

Number-line Estimation Task

The participants were asked to mark the blank number-line (with two endpoints of 0 and 100) where the target number should lie. Accuracy was defined as the absolute difference between the child's marked position and the correct position of the number (e.g., for the number 45, marking the line at 35 or 55 would result in a score of 10). The overall score was the mean of these differences across the 24 trials (Geary, 2018). An independent sample t-test was calculated to compare the difference between the mean of differences across the 24 trials in the pre-test and the mean of differences across the 24 trials in the post-test (see Table 4. 7). The results indicated that there was a significant difference between the mean of differences in the pre-test ($M= 18.87$, $SD= 1.48$) and the post-test ($M = 16.36$, $SD = 0.59$); $t (6) = 3.48$, $p< 0.05$.

Participants	Mean of the absolute difference between the child's placement and the correct position of the number in the pre-test	Mean of the absolute difference between the child's placement and the correct position of the number in the post-test
Aaden	20.833	17
Layla	17.33	15.71
Sincere	18.25	16.71
Azaleah	19.08	16.36
Mean (SD)	18.87 (1.48)	16.36 (0.59)
t-test (df)	3.48 (6)	

Table 4. 7: Participants' Pre-/Post-Intervention Scores in Number-Line Estimation Task

RESEARCH QUESTION THREE

The TEMI-AC total scores increased across the study and were maintained two and four weeks after the completion of the intervention for each participant. The level of maintenance data was larger than both baseline and intervention phases for all participants (see Table 4. 1). The level of maintenance data for Aaden, Layla, Sincere, and Azaleah were 59, 90, 72, and 101.5, respectively. Across the participants, the level of data increased at least 22 scores from the baseline phase to the maintenance phase. Results also demonstrated that students with a lower level in the baseline phase showed more improvement in the intervention phase and maintained the effect in the maintenance phase. Across all participants, the average level in baseline was 48.88 (SD = 27-96), which improved considerably during the maintenance phase (M = 80.63, SD = 19.51).

RESEARCH QUESTION FOUR

To examine the generalization effect of the intervention, the TEMA-3 was administered as a distal measure in both pre and post-intervention (see Table 4. 4). Cohen's d effect size was computed to assess the effect size of the TEMA-3 outcomes. Cohen (1969) suggested the following criteria for interpreting effect sizes: small effect (0.2 - 0.49), medium effect (0.5 - 0.79), and large effect (0.8 or greater). Table 4. 4 shows the TEMA-3 result in the pre- and post-intervention.

Participants	TEMA-3 Pre	TEMA-3 Post
Aaden	83	85
Layla	89	90
Sincere	80	93
Azaleah	87	97
Mean (SD)	84.75 (4.31)	91.25 (5.06)
Hedges' g	1.38	

Note. TEMA–3 = Test of Early Mathematics Ability–3: Raw score (Ginsburg & Baroody, 2003); SD = standard deviation

Table 4. 8: Participants' Pre-/Post-Intervention Mathematics Ability Scores and Effect Size

TEMA-3 was administered during the screening phase before the start of the intervention, and students' mathematics abilities scores were below the average, ranging from 83 to 89 (M = 85, SD = 4.40). The average pre-test score across all students was 84.75, which is within the below average category. The mean post-test score across participants was 91.25 (SD= 5.06), falling within the average range. The results indicated that the systematic strategic early mathematics intervention had a statistically significant effect (Hedges' g = 1.38) on the mathematical performance of students at risk for MD from pre- to post-intervention on TEMA-3.

RESEARCH QUESTION FIVE

To assess students' perspective about the early mathematics intervention, the investigator developed a social validity survey contained seven face-scale questions (i.e., 3: happy face, 2: natural face, and 1: sad face.), two open-ended questions to express their thought toward the intervention, and one yes/ no question to see if they would volunteer to participate in this intervention again. The investigator verbally asked the social validity questions and recorded their answers. The mean score for the face-scale questions was used to determine participants' perspectives toward the intervention. Table 4. 8 shows the results of the social validity survey for each participant.

All participants agreed that the intervention was beneficial, and they would volunteer to participate in the program again. They also reported that they enjoyed using manipulatives and being involved in multiple mathematics activities. They felt that the intervention helped them to understand mathematics better and do better in mathematics class. The teachers also reported students' improvement in the middle of the year benchmark test. Table 4. 9 shows students' performance in the school benchmark test at the beginning of the year, the middle of the year, and the end of the year.

Questions	Average Scale
I really like Mathematics.	3
I think mathematics is important.	2.75
The activities and lessons help me to do better in mathematics class.	3
The activities we did helped me to better understand mathematics concepts and skills.	3
The activities and lessons could help my classmates to do better in mathematics class.	3
Using different materials made the skills easier to understand.	3
I feel as though I was able to finish many of the problems independently on the worksheets.	3
I would volunteer to participate in this program again.	3
Favorite part of the intervention	Mathematics flashcards Practice sheets
Least favorite part of the intervention	None

Note: Rating scale: 1 = Disagree, 2 = Neutral, 3 = Agree

Table 4. 9: Social Validity Survey Results

SUMMARY OF CHAPTER

For research questions 1 and 3, the results of visual analysis and computation of the effect size (NAP) of the proximal measure (TEMA-AC) showed that the explicit, strategic early mathematics intervention was effective on the mathematics performance of first-grade students at risk for MD. All participants showed improvement in their mathematical skills and knowledge during the intervention phase and maintained intervention effects after two and four weeks.

For research question 2, the results demonstrated that there was a significant effect of the intervention on the participants' performance in the addition-strategy task (Hedge's $g = 1.61$), and number-sets tasks (Hedge's $g= 1.86$) improved significantly after receiving the intervention. There was also a significant difference between the mean of differences of the number-line estimation task in both the pre-test and post-test ($p< 0.05$). For research question 4, the TEMA-3 (distal measure) result in the pre/post-intervention demonstrated significant effects ($g = 1.38$) of the intervention on the overall mathematical performance of first-grade students with MD. For research question 5, all participants stated a positive perspective toward the intervention components and agreed that the intervention had a positive impact on their understanding of mathematics.

Chapter 5: Discussion

The purpose of this study was to examine the effects of a systematic strategic early mathematics intervention (e.g., explicit instruction and learning principles) on the proximal mathematics outcome (i.e., TEMI-AC), less proximal mathematics outcome (i.e., NSK tasks), and distal mathematics outcome (i.e., TEMA-3) of first-grade students at risk for mathematics difficulties. The intervention included the fundamental mathematical concepts and skills aligned with the first-grade Texas Essential Knowledge and Skills (TEKS), including addition/ subtraction combinations, number sequences, magnitude comparisons, and relationships of ten. The investigator delivered the intervention four days per week for 30 to 35 minutes sessions. Students received four lessons per week, Monday through Thursday. In total, 24 lessons were completed in six weeks.

Attaining basic mathematical competencies is a key element in later academic and career success (Baroody, Lai, & Mix, 2006; Morgan et al., 2009; and National Mathematics Advisory Panel [NMAP], 2008). Between 5% to 8% of school-age students are identified as having mathematics learning disabilities (Geary, 2011). Students begin to develop individual differences in early numeracy skills in the early years (Berch, Nava, Torquati, Sharp, & Richards, 2005). The achievement gap between students with MD and the average student increases as students move through the grades with more difficult curricula (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Jordan, Glutting, & Ramineni, 2010). Therefore, early mathematics intervention for students at risk for MD in primary grades is essential.

As early as kindergarten and first grade, achievement gaps can be identified between students at risk for MD and average students (Strand Cary et al., 2017). During early grades, several mathematical skills can be improved that build the foundation of mathematics learning in the future years (Sarama & Clements, 2009). These fundamental mathematical skills include basic number knowledge (cardinality, ordinality, one-to-one correspondence, and number estimation); number and operations in base-ten (single-digit and multi-digit calculations); and operations and algebraic thinking (solving addition and subtraction word problems). Thus, it is important that early mathematics intervention highlights these skills.

Previous studies demonstrated the positive effects of early mathematics intervention on the fundamental mathematics skills for students at risk for MD in early grades (Bryant et al., 2008; Casey et al., 2008; Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Jordan, Glutting, Dyson, Hassinger-das, & Irwin, 2015; Jordan, Glutting, Dyson, Hassinger-Das, et al., 2012; Klein et al., 2008; Sood & Jitendra, 2011). However, a significant subsample of students showed minimal response to the intervention or did not maintain the effects of the intervention. In this study, the investigator implemented a systematic strategic early mathematics intervention that involved explicit instruction and multiple learning principles (e.g., practice testing, scaffolding, and multiple representations) shown to be effective in both improving conceptual understanding and retaining of mathematical knowledge (Geary et al., 2017).

Participants in this study were identified as at risk for MD using a multiple-gating procedure that is a cost-effective stepwise screening mechanism for identifying eligible participants (Loeber, 1990; Loeber et al., 1984). In this study, the first gate involved using the result of universal screening measures at school (Pearson Education End of Year test; Envisions, 2014), a relatively inexpensive screening conducted with all school students (Loeber, 1990). The second screening gate was conducted only with a pool of possible at-risk participants identified in the first step (i.e., having below 70 percent accuracy on the test as designated by DMAC Solutions, Education Service Center in 2018). The second gate of screening was more elaborate and time-consuming than the first screening utilizing the TEMA-3 (Ginsburg & Baroody, 2003) and identified students whose scores ranked below the 30th percentile. In the last gate, four students who had the lowest scores and met the inclusion and exclusion criteria were identified to participate in the study. This procedure aligned with previous studies, for example, Catts et al. (2001) and Gilbert et al. (2012). Moreover, Purpura et al. (2015) identified students at risk of later mathematics difficulties based on TEMA-3 scores of 90 or below using sensitivity (i.e., the proportion of students correctly classified as at risk) and specificity (i.e., the proportion of students correctly classified as not at risk). Previous studies suggested to overidentify students at risk of later difficulties and then remove false positives through subsequent assessment methods than to under-identify and miss children at risk of mathematics difficulties and in need of further instruction (Purpura & Logan, 2015).

To examine the effects of the systematic strategic early mathematics intervention, the investigator implemented a multiple baseline design across participants. During the baseline phase, the TEMI-AC was administered to the participants twice a week at approximately the same time during the school day when future intervention sessions were implemented. During the intervention phase, participants attended four intervention sessions (Monday through Thursday) and one review session (Fridays) per week for six weeks. Five research questions were examined in this study:

1. Does the early mathematics intervention result in improved performance on the TEMI-AC, which is a weekly proximal measure of mathematics?
2. Does the early mathematics intervention result in improved performance on the NSK task, which is less proximal to the intervention?
3. Does the early mathematics intervention result in improved performance on the TEMA, which is a distal mathematics measure?
4. Are the effects of the intervention maintained 2- and 4-weeks post-intervention based on performance on the TEMI-AC?
5. What is the students' perspective regarding this early mathematics intervention?

This chapter discusses the results concerning the above research questions. The first part of this chapter is a discussion of the results for research question one and research question three. Students' outcomes on TEMI-AC as a proximal measure and the maintenance effect of the intervention were discussed. The results for students' outcomes on NSK tasks as a less proximal measure were discussed in the next section. The section following this explores research question four, which is a discussion on the generalization of the students' outcomes measured through TEMA-3 as a distal measure. Finally, students' perspective toward the intervention measured through a social validity form was discussed. The chapter concluded with a discussion of limitations of the study, recommendations for future areas of research, and implications for practice.

RESEARCH QUESTION ONE

To examine the effect of the early mathematics intervention on the mathematics performance of first-grade students at risk for MD, both visual analysis and proximal effect sizes (i.e., of visual data) were employed. The results of the visual analysis demonstrated that there was a causal relationship between the systematic strategic mathematics intervention and participants' early mathematical knowledge and skills. All four participants demonstrated a lower level of mathematics performance during the baseline phase, but their level improved significantly from the baseline to the intervention phase (Aaden: 53, change of level: 39.43; Layla: 87.08, change of level: 25.36; Sincere: 69.83, change of level: 28.17; Azaleah: 97.33, change of level: 18.75). The level of participants' mathematical performance increased after introducing the intervention, and all participants maintained the intervention effects two and four weeks following the intervention phase. However, students who started lower than others in baseline showed larger positive outcomes in intervention and maintenance phases. For example, Aaden, who had the lowest level in the baseline (53), showed the highest change of level (39.43) in the intervention phase. The result suggested that students with mathematics difficulties benefit more from the implementation of early mathematics interventions.

Across participants' total scores on bi-weekly TEMI-AC, the visual data effect sizes (i.e., NAP) were large, showing the significant large effect of systematic strategic early mathematics intervention on the mathematical skills of all participants (Parker &

Vannest, 2009).The NAP of 100% for all participants showed that from baseline to the intervention phase, data demonstrated a strong improvement (Parker & Vannest, 2009).

According to both the visual analysis and effect sizes computed, it was indicated that explicit, systematic strategic early mathematics intervention was effective for supporting mathematical knowledge and skills of students at risk for MD. Previous intervention studies utilizing early explicit mathematics intervention have shown consistent, positive effects on mathematics performance of students at risk for MD (e.g., (Clarke et al., 2011; Dyson et al., 2013; R. Gersten et al., 2015; Hassinger-Das et al., 2015; Jordan et al., 2009; Klein et al., 2008). Bryant et al. (2016) indicated that even the most struggling students could benefit from small group intervention that is intensive, strategic, and explicit. Thus, the instructional design features of the intervention (e.g., explicit, direct, and engaging instruction; instructional time, hands-on materials, flash-cards, worksheets, and activities) could be a possible factor that contributed to the effectiveness of the intervention (Bryant et al., 2016).

In addition to the instructional design features, learning principles that were embedded in the design of the intervention could contribute to the maintenance effects of the intervention. The systematic strategic early mathematics intervention included various learning principles (e.g., practice testing, interleaving practice, scaffolding, feedback, error reflection, multiple representations, and worked examples) recommended by previous researchers (Dunlosky et al., 2013; Geary et al., 2017). Smith, Holliday, and Austin (2010) reported that asking explanatory questions (e.g., why) during the lesson can encourage students to integrate new information from a lesson with their prior knowledge. Rohrer, Dedrick, and Burgess (2014) indicated that interleaving practice could help students build a strong relationship between problem types and appropriate solution strategies (Rohrer et al., 2014). Students who received practice on a set of previously learned relevant concepts performed better at the post-test (Pellegrino, 2012).

Regarding scaffolding, Fuchs, Fuchs, and Compton (2012) noted that scaffolding through short-term validated intervention creates a strong foundation for students at risk for MD to experience long-term success with their mathematics learning (Fuchs et al., 2012). Barbieri and Booth (2016) have found that studying and explaining errors can help students with low and high prior knowledge in mathematics to learn mathematics (Barbieri & Booth, 2016). Also, previous researchers suggested that integrating concrete and abstract representations can provide considerable benefits for students at risk for MD because concrete representations support conceptual understanding (e.g., Kaminski, Sloutsky, & Heckler, 2005), and abstract representations support the transfer of knowledge and skills (Bock et al., 2011).

Additionally, all participants who qualified for this study come from low SES backgrounds. It is important to note that children from low SES backgrounds have been shown to have difficulties in early numeracy knowledge and skills (e.g., magnitude comparison, addition/subtraction combination, and counting) compared to their peers from high socioeconomic backgrounds (Griffin, Case, & Siegler, 1994; Jordan, Kaplan, Nabors Oláh, & Locuniak, 2006). The positive outcomes of the early mathematics intervention were similar to previous research studies suggesting that early mathematics intervention positively affects the mathematical abilities of children from economically disadvantaged backgrounds (Clements & Sarama, 2008; Whyte & Bull, 2008).

RESEARCH QUESTION TWO

Research question 2 examined the effect of systematic strategic early mathematics intervention on the early numeracy knowledge and skills through NSK measure, which is less proximal to the intervention. The results showed that participants' performance improved significantly in the addition-strategy task (Hedges $g = 1.61$) and the number-sets task (Hedges' $g = 1.86$) after receiving the intervention. There was also a significant difference between participants' performance in the number-line estimation task in pre-test and post-test ($p < 0.05$).

Regarding the addition-strategy task, there was a 6-point scale that reflected both accuracy and sophistication of the strategy students used: 1 = error in using the retrieval, fingers, or decomposition strategy; 2 = error in using a counting strategy, whether finger or verbal counting; 3 = correct use of the max or sum counting strategy; 4 = correct use of the min counting strategy; 5 = correct use of retrieval-related strategies (fingers and decomposition); and 6 = correct retrieval. The most frequent point scales that students used to solve the addition problem included (a) error in using the retrieval, fingers, or decomposition strategy; (b) error in using a counting strategy; and (c) correct use of the max or sum counting strategy. Previous studies (e.g., Geary, 2011; Montague, 1997; Rosenzweig, Krawec, & Montague, 2011) also found that students with learning difficulties often use developmentally immature, inefficient strategies (e.g., sum counting strategy, error in using fingers) rather than more mature strategies (e.g., correct retrieval) (Geary, 2007; Sherin & Fuson, 2005). Also, among the accurate answers, the most common strategy that students used in the pre-test were the correct use of the max or sum counting strategy. However, among the accurate answers, the most common strategies that students used in the post-test were the correct use of the min counting strategy and correct retrieval.

These results are similar to prior research studies in early mathematics intervention for students with MD (e.g., Dyson, Jordan, & Glutting, 2013b; Jordan, Glutting, Dyson, Hassinger-Das, et al., 2012; Klein et al., 2008). Locuniak and Jordan (2008) suggested that students with MD struggle to develop sufficient number sense knowledge and skills required to facilitate later conceptual understanding of mathematical concepts (Locuniak & Jordan, 2008). Number sense is a predictor of mathematics achievement at the conclusion of first-grade (Jordan et al., 2009).

The findings of this study were consistent with previous research; that is, explicit, strategic instruction could be an effective instructional approach for improving mathematical skills and promoting the use of mature, efficient strategies in mathematics (e.g., Iseman & Naglieri, 2011; Tournaki, 1993; Van Houten, 1993; Van Luit & Naglieri, 1999; Woodward, 2006). Scholars (e.g., Cary et al., 2017) have found that early mathematics intervention for at-risk students has a positive, statistically significant effect on promoting students' mathematical proficiency with whole number concepts and skills. Clarke et al. (2017) also reported that explicit teacher modeling, deliberate practice, multiple representations of mathematics, and academic feedback could promote early numeracy skills (Hedges's $g = 0.755$), and proficiency in early number sense (Hedges's g

= .52). Kroesbergen and Van Luit (2003) suggested that using explicit models for teaching conceptual understanding, such as instruction used in this study, could have successful outcomes for students at risk for learning disabilities (Kroesbergen & Van Luit, 2003).

In addition to explicit instruction, using interleaving practice at the beginning of each intervention session as a warm-up activity could be another possible factor that contributed to participants' improved addition strategy tasks. This result was consistent with previous studies. For example, Schutte et al. (2015) indicated that interleaving practice could be beneficial for students in the retrieval of both addition and multiplication facts. Rohrer, Dedrick, and Burgess (2014) also suggested that interleaving practice can help students in choosing appropriate solution strategies to solve mathematics problems (Rohrer, Dedrick, & Burgess, 2014). Interleaving practice also provides more opportunities for students to identify errors and refine their knowledge on several different addition strategies (Li et al., 2012).

The results of this study showed that explicit, systematic strategic intervention could be an effective instructional approach for teaching mature and efficient mathematics strategies to first-grade students at risk for MD (e.g., Iseman & Naglieri, 2011; Van Luit & Naglieri, 1999; Woodward, 2006). Overall, teaching addition/subtraction combination strategies help students select effective strategies or use the strategies effectively and efficiently (Swanson, 1989).

RESEARCH QUESTION THREE

The results of the maintenance effect of the systematic strategic early mathematics intervention demonstrated that all participants' TEMI-AC scores improvement maintained two and four weeks after the completion of the intervention phase. Therefore, the systematic strategic early mathematics intervention was effective for students at risk for MD, even after removing the intervention. The finding was consistent with previous research (e.g., Bryant et al., 2008; Bryant et al., 2011; Dennis, Bryant, & Drogan, 2015; Dennis, Sorrells, & Falcomata, 2014), showing the explicit, strategic early mathematics intervention is effective in both improving mathematical skills and maintaining the skills over time. All participants maintained the effect of the intervention over time (M = 80.63, SD = 18.85, range of between 55 and 102). The learning principles embedded in the intervention were possible variables that accounted for improving mathematics learning and maintaining the effects after the intervention. Dunlosky et al. (2013) reported that spreading out learning opportunities may result in better long-term retention of information (Dunlosky et al., 2013). Carpenter (2009) also suggested that practice testing can improve retention by triggering elaborative retrieval processes because it involves a search for long-term memory that activates related information (Carpenter et al., 2009).

RESEARCH QUESTION FOUR

Research question 4 was related to participants' mathematics performance on a distal measure (TEMA-3) from pre-test to post-test. The finding suggested that the explicit, strategic early mathematics intervention is effective in promoting mathematics performance of first-grade students at risk for MD on the distal measure. TEMA-3 is a standardized assessment that measures students' formal and informal mathematical skills, including numeral literacy, number facts, and understanding of place value and base 10 number system. Participants showed statistically significant improvement from pre-test to post-test ranging from 1 to 13 standardized scores gain as measured by the TEMA-3. The results revealed that early mathematics intervention in first-grade is essential in order to close the gaps between students at risk for MD and their typically developing peers. Rasanen, Salminen, Wilson, Aunio, and Dehaene (2009) indicated that without appropriate early intervention, the gap between the high achievers and at-risk students continues to expand and can have a long-term effect in students' future academic success (Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009). Previous studies have shown consistent, positive effects of early mathematics intervention on mathematics standardized measures (Hassinger-Das et al., 2015; Jordan, Glutting, Dyson, Hassinger-Das, et al., 2012; Sood & Jitendra, 2011).

Regarding the effectiveness of the early mathematics intervention, the findings are consistent with the literature showing that explicit, systematic intervention is an effective approach for young students at risk for MD. Bryant et al. (2014) reported a significant effect of early mathematics intervention on the mathematics performance of second-grade students with severe MD measuring through KeyMath-3. Researchers have incorporated various forms of early intervention and found positive effects on students' mathematics performance. For instance, Clarke et al. (2017) found that Tier 2 mathematics intervention has statistically significant effects on TEMA-3 (Hedges's g = 0.25). Dyson et al. (2011)

conducted a number sense intervention for kindergartners from low-income families and found that the intervention group made meaningful gains relative to the control group in both immediate and delayed post-tests on a standardized test of mathematics calculation measure. Also, in their meta-analysis, Wang et al. (2016) reported that early mathematics interventions showed small to moderate effect size using standardized mathematics outcome measures.

RESEARCH QUESTION FIVE

Research question 5 examined the perspective of participants about the explicit, strategic early mathematics intervention. The findings revealed that, on average, students had a positive perspective toward the intervention. All participants indicated that the activities and lessons helped them perform better in their mathematics classes. Participants also believed that using different materials made the mathematics skills easier to understand. All students expressed high levels of interest in participating in the program in the future. They also believed that the activities and lessons would help their classmates to do better in mathematics. All participants but one stated that learning mathematics is important. The participants' most favorite parts of the intervention were the flash card activities, hands-on material, and independent practice worksheets. The results were aligned with previous studies showing positive perceptions of participants about the program's effectiveness and benefits (e.g., Calhoon, Al Otaiba, Cihak, King, & Avalos, 2007; Jitendra et al., 2004)

EDUCATIONAL IMPLICATIONS

The results of this study demonstrate multiple implications for practice. First, the findings suggest that systematic strategic early mathematics intervention using explicit instruction and multiple learning principle could be effective in teaching mathematics to students at risk for MD. The results were similar to the findings of previous mathematics intervention research for early grades (e.g., Clarke, Doabler, Kosty, Nelson, & Smolkowski, 2017; Clements & Sarama, 2008; Klein et al., 2008). The intervention consisted of the explicit instructional components (e.g., warm-up, modeled practice, guided practice, independent practice, check for understanding, and error correction) and multiple learning principles for teaching mathematics (e.g., elaboration interrogation, practice testing, interleaving practice, scaffolding, feedback, and multiple representations). Teachers can use these instructional methods in the mathematics classroom to improve mathematical learning in early grades for students at risk for MD (Bryant et al., 2008; Fuchs, Fuchs, and Hollenbeck, 2007). Many of the recommended principles have been embedded in mathematics instruction in countries with traditionally higher mathematics achievement. For example, teachers in Hong Kong and Japan make more connections between abstract and concrete representations (Richland, Zur, & Holyoak, 2007); explanatory questioning is more common in Japanese textbooks, and error reflection is critical for learning in Japanese classrooms (Booth et al., 2017; Mayer, Sims, & Tajika, 1995). The result of this study and the fact that these principles are prominent in countries that outperform the U.S. suggest that these principles may also be useful in teaching mathematics in U.S. classrooms.

All participants in this study were from low SES background, and the results showed a significant effect of early mathematics intervention in this sample which aligned with previous research on early mathematics intervention for this sample (e.g., Jordan, Glutting, Dyson, Hassinger-das, & Irwin, 2012; Klein et al., 2008; Siegler & Ramani, 2008). The result of this study suggests that early mathematics intervention can be academically effective for students from low SES backgrounds.

LIMITATIONS OF THE STUDY

Several limitations need to be considered when reviewing the results of this study. The first limitation is that the explicit, strategic early mathematics intervention had multiple components. It included the components of explicit instruction (R. Gersten et al., 2009; Wang et al., 2016) and consisted of several learning principles for teaching mathematics (Dunlosky et al., 2013; Geary et al., 2017). Although the intervention was effective in improving mathematical knowledge and skills of students at risk for MD, it is not clear which intervention component is more effective for each mathematics concept. For example, the learning principles may vary in terms of their effectiveness for simpler versus more complex mathematical content.

Second, although all progress monitoring measures were double-coded by another trained doctoral student, the investigator implemented the intervention session and administered all proximal measures during the intervention. The result cannot be generalized to the effect of teachers or school interventionists implementing the intervention.

Third, this study was conducted with native English speakers who live in a large, urban city. All participants in the study were identified as at risk for learning difficulties and also were economically disadvantaged. The results of this study cannot be generalized to English learners, students from different racial/ethnic backgrounds, or students who live in suburban or rural areas.

FUTURE RESEARCH

First, the finding suggests that the use of multiple instructional components does not allow researchers to determine which principles could be more important than others. Although the learning principles, such as interleaving practice and feedback, were included as part of the design of mathematics interventions, these principles have not been explicitly manipulated to determine whether or not such principles are effective for improving mathematical learning outcomes in young children at risk of MD. Perhaps struggling learners stand to benefit the most from the stringent application of such principles in the instructional design. However, the researchers and educators have not purposefully tested these principles in applied intervention settings. So, more research needs to be done in this area.

Second, future research needs to examine the effect of teachers or school interventionists implementing this intervention to allow scaling up the intervention, which would involve the integration of the components into the routines of the teaching practices (Odom, 2008).

Finally, although the demographics of the participants represents part of the students' demographics who are identified as at risk, there are still many ELs students and students with different demographic characteristics who may respond to the intervention differently. Future researchers may consider examining the intervention for students with different demographics and ELs.

Appendix A: Schedule, Scope and Sequence

Week 1		
Day 1: Monday	Relationships of 10	0–50
Day 2: Tuesday	Progress monitoring Addition/Subtraction Combinations	+/– 1, +/– 0, n – n
Day 3: Wednesday	Magnitude Comparison Number Sequences	0–50
Day 4: Thursday	Addition/Subtraction Combinations	+/– 1, +/– 0, n – n
Day 5: Friday	Progress monitoring Review	
Week 2		
Day 1: Monday	Relationships of 10	0–50
Day 2: Tuesday	Progress monitoring Addition/Subtraction Combinations	+/– 2, +/– 3
Day 3: Wednesday	Magnitude Comparison Number Sequences	0–50
Day 4: Thursday	Addition/Subtraction Combinations	+/– 2, +/– 3
Day 5: Friday	Progress monitoring Review	
Week 3		
Day 1: Monday	Relationships of 10	0–50
Day 2: Tuesday	Progress monitoring Addition/Subtraction Combinations	Fact families strategy
Day 3: Wednesday	Magnitude Comparison Number Sequences	0–50
Day 4: Thursday	Addition/Subtraction Combinations	Fact families strategy
Day 5: Friday	Progress monitoring Review	

Week 4		
Day 1: Monday	Relationships of 10	0–50
Day 2: Tuesday	Progress monitoring Addition/Subtraction Combinations	Doubles
Day 3: Wednesday	Magnitude Comparison Number Sequences	0–50
Day 4: Thursday	Addition/Subtraction Combinations	Doubles
Day 5: Friday	Progress monitoring Review	
Week 5		
Day 1: Monday	Relationships of 10	50–99
Day 2: Tuesday	Progress monitoring Addition/Subtraction Combinations	Doubles + 1
Day 3: Wednesday	Magnitude Comparison Number Sequences	50–99
Day 4: Thursday	Addition/Subtraction Combinations	Doubles + 1
Day 5: Friday	Progress monitoring Review	
Week 6		
Day 1: Monday	Relationships of 10	50–99
Day 2: Tuesday	Progress monitoring Addition/Subtraction Combinations	Doubles + 1 and related
Day 3: Wednesday	Magnitude Comparison Number Sequences	50–99
Day 4: Thursday	Addition/Subtraction Combinations	Doubles + 1 and related
Day 5: Friday	Progress monitoring Review	

Appendix B: Student Social Validity Scale

SID: ____________________________

School: ___________________________ Date: _______________________________

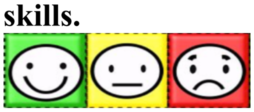
Please read each statement and circle the comment that reflects your response.

I really like Mathematics.

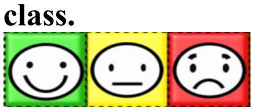
I think Mathematics is important.

The activities and lessons help me to do better in Mathematics class.

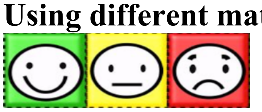
The activities we did helped me to better understand Mathematics concepts and skills.



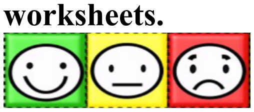
The activities and lessons could help my classmates to do better in Mathematics class.



Using different materials made the skills easier to understand.



I feel as though I was able to finish many of the problems independently on the worksheets.



I would volunteer to participate in this program again YES or NO

Appendix C: Fidelity Check List

Early Mathematics Intervention Fidelity Checklist

2018

A. Session Info

Interventionist		**Time**	
School		**Lesson**	
Date		**Observer**	
Type of fidelity check	☐ Live observation ☐ Audio ecording		

B. Interventionist Competence

Elements	Score 1	Score 2	Score 3	
Shows good command and knowledge of subject matter	1	1	1	Percentage
Demonstrates breadth and depth of mastery				
Follows script, as necessary				
Has all needed materials prepared for the lesson				
Has few long pauses in execution of lesson				
Uses timer/recorder to monitor time spent on the session				
sum	1	2	3	40

D. Explicit Instruction

Elements	Score 1	Score 2	Score 3	
Provides sufficient cumulative review both before, during, and after instruction.				Percentage
Provides support to promote learning when concepts and skills are being first introduced to students.				
Watches and/or listens to student responses (of any form) to ensure that all students are mastering the skills that are being presented.				
Uses methods to promote active student response.				
Provides good models and uses think-alouds to make thinking visible and easier for students to follow.				
sum	0	0	0	0

D. Practice opportunities

Elements	Score 1	Score 2	Score 3	
Provides varied opportunities for practice that are linked to the instruction				Percentage
Provides immediate, corrective and descriptive comments/suggestions for both correct and incorrect responses.				
sum	0	0	0	0

E. Worked Example and self explenation

Elements	Score 1	Score 2	Score 3	
Prompts students to explain the information to themselves.				Percentage
asks students to explain a combination of correct and incorrect examples				
Helps students to recognize and accept when they have chosen incorrect procedures				
Draws students' attention to the particular features in a problem that make the procedure inappropriate.				
For correct examples, promotes students to indicate "why" the step was right				
sum	0	0	0	0

Indicators	Score 1	Score 2	Score 3	
Provides quality of treatment				Percentage
Makes the students feel valued and welcome				
Is responsible to the student's behaviors				
Keeps the lesson fun and engaging				
Corrects and offers positive feedback to the students				
Acknowledge the student's achievements and set new goals				
Uses time wisely				
Demonstrates leadership ability				
Maintains discipline and control				
sum	0	0	0	0

Total	
Strengths observed	
Suggestions for improvement	
Overall impression of teaching effectiveness	

Number Line: Find It!

Number Sequences

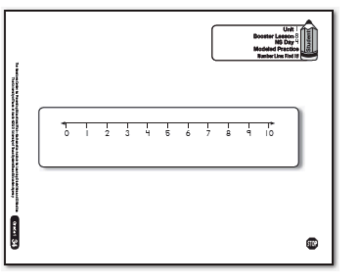
Objective: **The student will be able to locate numbers on a number line.**

Instructional Content: 0–50

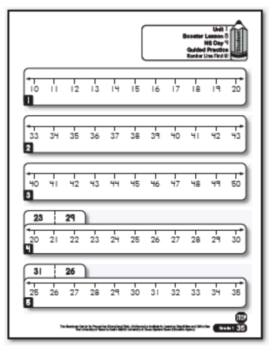
Vocabulary: Number, number line

Materials: Teacher Master, pp. 34–36; wipe board

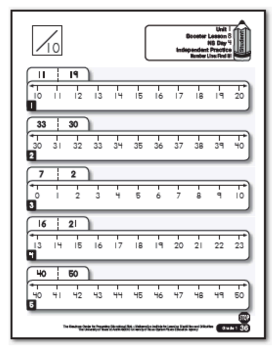
Modeled Practice



Guided Practice



Independent Practice



6 Time:

Set the timer for 6 minutes. Spend the majority of the time on Guided Practice.

Error Diagnosis and Correction

The student skips numbers or counts inaccurately: tell the student to count more slowly and to touch each number as he or she counts.

Preview

Today we will look at the number line and locate numbers.

Modeled Practice
(My Turn, Your Turn)

1. Distribute a Modeled Practice sheet to each student. Point to each number as you say it. Write "3" on the wipe board.

My Turn: I have a number line. It starts by showing the number 0 and shows numbers up to 10.

Your Turn: What number? *(point to the wipe board; 3)*

I want to find the number 3 on the number line.

We start at 0 and count up. Ready? Count. 1, 2, 3.

We found the number 3 on the number line!

Circle it.

Guided Practice
(Our Turn)

2. Distribute a Guided Practice sheet to each student. Write "12" on the wipe board.

What number? *(12)*

Find the number 12 on the number line.

How can we find the number 12?

Start at the number 10 and count up together. Ready? Count. 10, 11, 12.

Circle it.

3. Use the same number line on the Guided Practice sheet; write "18" on the wipe board.

Guided Practice
(continued)

What number? *(18)*

Find the number 18 on the number line.

How can we find the number 18? Counting all the way from the number 10 will take a long time. Let's count back from the number 20.

Count back together. Ready? Count. 20, 19, 18.

Circle it.

❹ Using the same number line on the Guided Practice sheet, write "8" on the wipe board (a nonexample).

What number? *(8)*

Find the number 8 on the number line.

Where is 8? *(it is not on this number line; it is less than the numbers on this number line; 8 is smaller or less than 10)*

It is not on this number line.

❺ Repeat steps 2–4 for the next 2 number lines on the Guided Practice sheet. For each number line, write a number on the wipe board, and then tell students to read the number, find it on the number line, and circle it. For each number line, use 1 example and 1 nonexample.

Error Diagnosis and Correction

A student cannot find a number on the number line: point to the number, say the number, and count up the number line to it.

❻ For the last 2 number lines on the Guided Practice sheet, use the following language:

Look at the 2 numbers in the box and circle them on the number line.

What number?

Circle it on the number line.

Independent Practice/ Progress Monitoring
(Your Turn)

Time:

Set the timer for 2 minutes. For the first minute, have students complete the Independent Practice sheet.

❶ **For 1 minute:** Distribute an Independent Practice sheet to each student and tell the students to complete as many items as possible.

> **You will have 1 minute to look at the numbers in the box and circle them on the number line.**

❷ **For remaining time:** Go through the items with the students, telling them the correct answers. They should put a check mark *(√)* by correct answers and should correct any errors.

Note to Teacher:

Score 1 point for each correctly circled number on the number line.

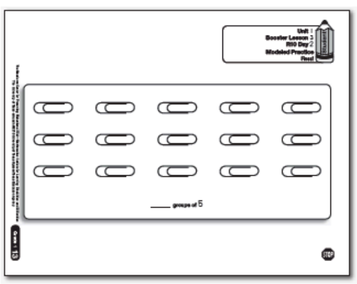
❸ Record their scores as the number correct / total number possible.

Fives!

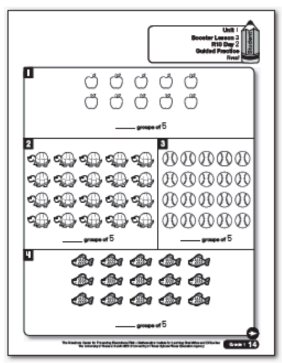
Relationships of 10

Objective:	**The student will be able to group concrete and pictorial objects into groups of five and write the number that represents the total amount.**
Instructional Content:	Groups of 5 totaling 0 to 50
Vocabulary:	Groups
Materials:	Teacher Master, pp. 13–16; paper clips (or other common objects such as teddy bear counters, crayons, or stones; T&S; 20 of each)

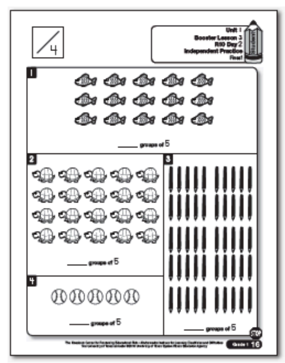
Modeled Practice



Guided Practice



Independent Practice



Time:

Set the timer for 12 minutes. Spend the majority of the time on Guided Practice.

Preview

We will group paper clips to help us count them and then write the number that tells how many groups there are.

Note to Teacher:

Select an object that is familiar to students; paper clips will be used in this lesson.

Modeled Practice
(My Turn, Your Turn)

❶ Place 15 paper clips on the table in a row.

My Turn: I have a bunch of paper clips. *(point to the row)*

I need to count them quickly.

Your Turn: Count the paper clips 1 by 1. Ready? Count. 1, 2 … 15.

Error Diagnosis and Correction

A student has difficulty circling groups of 5: tell the student to count out loud and cross out each object before circling a group of objects.

❷ Distribute a Modeled Practice sheet and 15 paper clips to each student. Place the paper clips on top of the pictorial representations on the sheet. Tell the students to count the groups of 5. Write the numeral for the total number of groups on the Modeled Practice sheet and tell the students to read the numeral.

My Turn: I put my paper clips in groups of 5 to count them quickly.

Your Turn: Ready? Help me count out 5 paper clips for each group. Count. 1, 2 … 5. *(repeat until there are 3 groups of 5 paper clips)*

How many groups of paper clips?

Count.1 group of 5 paper clips, 2 groups of 5 paper clips, 3 groups of 5 paper clips.

❸ Write "3" groups of 5 on the Modeled Practice sheet.

My Turn: I write the number 3. 3 groups of 5 paper clips.

Your Turn: Write the number 3 to show how many groups of 5 paper clips we made.

Error Diagnosis and Correction

A student has difficulty counting groups of 5: tell the student to count out loud and cross out each circled group.

Guided Practice
(Our Turn)

❹ Distribute 20 objects to each student. Tell students to make groups of 5, count the groups, and say the number of groups out loud.

> **Put your paper clips in groups of 5. Ready? Count. 1, 2 … 5.**
>
> **How many groups of 5? Count. 1 group of 5, 2 groups of 5, 3 groups of 5, 4 groups of 5.**

❺ Distribute the Guided Practice sheets. Tell students to circle groups of 5 objects for each item and state how many groups of 5 there are for each item. Obtain both choral and individual responses. Use the following language for the first item:

> **How many groups of apples are there?**
>
> **Circle groups of 5 apples.**
>
> **Ready? Count. 1, 2 … 5. Circle it.**
>
> **Ready? Count. 1, 2 … 5. Circle it.**
>
> **How many groups of 5 apples did we circle?**
>
> **Write it.**

Independent Practice/ Progress Monitoring
(Your Turn)

2 Time:

Set the timer for 2 minutes. For the first minute, have students complete the Independent Practice sheet.

❶ **For 1 minute:** Distribute an Independent Practice sheet to each student and tell students to complete as many items as possible.

> **You will have 1 minute to circle groups of 5 objects. Then write the total number of groups you circled for each item.**

❷ **For the remaining time:** Go through the items with the students, telling them the correct answers. They should put a check mark *(√)* by correct answers and should correct any errors.

Note to Teacher:

Score 1 point for each correctly written number of groups of 5.

Appendix E: TEMI-AC Samples – Weekly Progress Monitoring

TEMI K

Texas Early Mathematics Inventories

Aim Check

FORM A

Student Name

____

MC ______
NID ______
NS ______
QR ______

Total ____

13	4	2	10	1	2	12	4
4	18	3	3	3	1	14	1
10	7	6	13	5	4	2	12
5	15	3	11	7	7	4	8
14	4	15	3	7	5	4	6
9	5	10	9	7	9	14	14
6	8	14	7	11	9	10	12
2	5	13	10	15	13	11	15

7 8 ___	
6	9
11	10

5 6 ___	
7	17
9	4

4 5 ___	
6	3
14	13

___ 5 6	
2	7
3	4

1 ___ 3	
4	2
0	5

4 ___ 6	
3	7
17	5

8 9 ___	
7	11
10	5

5 ___ 7	
10	6
8	4

___ 1 2	
0	2
4	9

1 2 ___	
4	3
0	5

6 ___ 8	
14	10
9	7

___ 10 11	
1	9
5	12

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